REPORT ON "LARGE PROPERTIES AT SMALL CARDINALS" BY LAURA FONTANELLA

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The thesis "Large properties at small cardinals" is concerned with versions of a combinatorial property known as the *tree property*. The classical version of the tree property asserts of an uncountable regular cardinal κ that every κ-tree has a cofinal branch, that is to say there are no κ-Aronszajn trees. This is a property of large cardinal type (the inaccessible cardinals with the tree property are exactly the weakly compact cardinals), which can consistently hold at small cardinals. It is a particularly interesting property because (in contrast with principles such as stationary reflection or the existence of saturated ideals) it is fragile under small forcing and closely tied to the behaviour of the continuum function. The tree property has been the object of much research and the catalyst for several advances in forcing.

The main subject of Fontanella's work is versions of the tree property in which κ is replaced by $P_\kappa\lambda$ for $\lambda > \kappa$, notably the *strong* (κ, λ)*-tree property* and the *super* (κ, λ)*-property*. These properties have been known and studied for some time, but have recently come to more prominence because of some striking results by Matteo Viale and Christoph Weiss, involving new ideas for lower bounds on the strength of principles such as the Proper Forcing Axiom. Again they are closely tied to classical large cardinal properties, namely for κ inaccessible we have that "for all λ the strong (κ, λ)-tree property holds" is equivalent to κ being strongly compact, while "for all λ the super (κ, λ)-tree property holds" is equivalent to κ being supercompact.

The thesis contains three main results, each of them quite striking and marking an advance in our knowledge of these intriguing properties:

- It is consistent that for every n such that $1 < n < \omega$ and every λ the super $(\aleph_n, \lambda)$-tree property holds. In fact Fontanella proves that this holds in a model due to me and Matt Foreman (This result was proved independently by my student Spencer Unger at about the same time, incidentally to his investigation of other reflection properties true in this model).

- If ν is a singular limit of strongly compact cardinals then the strong (ν^+, λ)-tree property holds for all λ.
- It is consistent that the strong $(\aleph_{\omega+1}, \lambda)$-tree property holds for all λ.

The proofs are ingenious and display Fontanella's excellent grasp of some highly technical machinery involving large cardinals and iterated forcing. The exposition is self-contained and is uniformly excellent, The thesis amply fulfils the requirements for a doctoral thesis. I would go further and say that Fontanella's thesis is one of the best recent doctoral theses in set theory, and her doctorate should be awarded with whatever special honours are available.