First order synthesis for data words revisited

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Motivation

- We want an **unbounded number of agents**...
  - processes
  - computers in a network
  - drones

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- ...to satisfy some **specification**

System and **Environment**, playing actions (a and b for System, c and d for Environment) in turn on shared or proper agents:

(1,a)(8,b)(7,d)(4,c)(6,a)(6,c)(7,a)(6,d)(2,b)(7,d)(7,a)

Executions: finite or infinite **data words**

`(1,a)(8,b)(7,d)(4,c)(6,a)(6,c)(7,a)(6,d)(2,b)(7,d)(7,a)`

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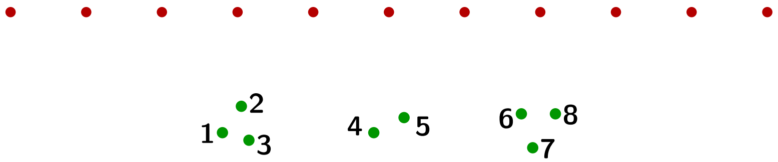
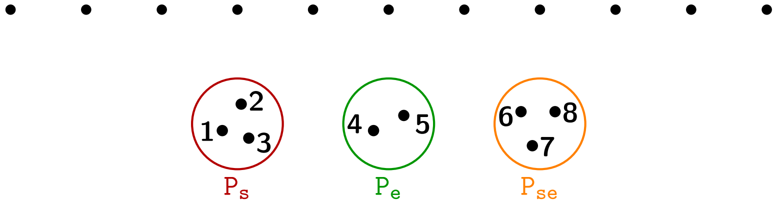


- One element for each position
- One element for each agent

Executions: finite or infinite **data words**

`(1,a)(8,b)(7,d)(4,c)(6,a)(6,c)(7,a)(6,d)(2,b)(7,d)(7,a)`



- Three unary relations P_s, P_e and P_{se} to denote ownership of the agents

Executions: finite or infinite **data words**

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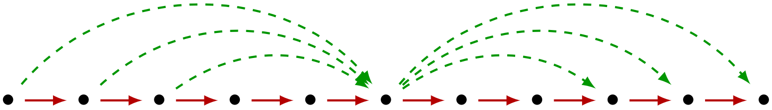
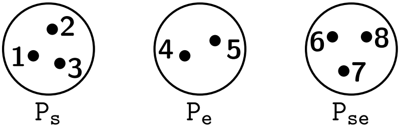
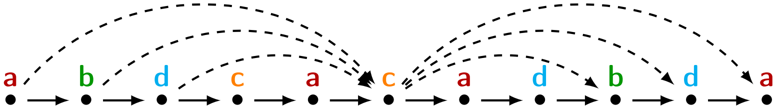




- A binary relation $+1$ between successive positions
- A binary relation $<$ for its transitive closure

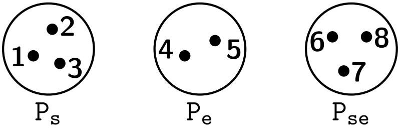
Executions: finite or infinite **data words**

(1,a)(8,b)(7,d)(4,c)(6,a)(6,c)(7,a)(6,d)(2,b)(7,d)(7,a)



$\mathtt{A}_s = \{a, b\}$

$\mathtt{A}_e = \{c, d\}$



- A unary relation for each action

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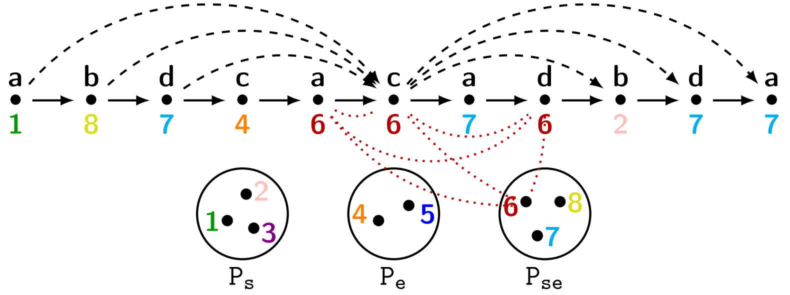

- An equivalence relation $\sim$ with a class for each agent

Specification language

Fragment of first-order logic , with a subset of the binary predicates

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- $\mathsf{FO}^2[\sim, <, +1]$

Specification language

$\overbrace{\text{Fragment of first-order logic}}^{\text{two variables}}$, with a subset of the binary predicates

- $\mathsf{FO}^2[\sim, <, +1]$

Specification language

Fragment of first-order logic , with a $\overbrace{\text{subset of the binary predicates}}^{\text{all predicates}}$

- $\mathsf{FO}^2[\sim, <, +1]$

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Every agent requesting a resource eventually gets it:

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Every agent requesting a resource eventually gets it:

$\forall x,\ \mathtt{req(x)} \rightarrow\ \exists y,\ y \sim x\ \wedge\ y > x\ \wedge\ \mathtt{gets(y)}$

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- $\mathrm{FO}[\sim]$

Specification language

$\overbrace{\text{Fragment of first-order logic}}^{\text{no restriction}}$, with a subset of the binary predicates

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> *Every System agent requests at most twice a resource:*

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- $\mathrm{FO}[\sim]$

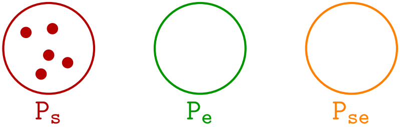
  Every System agent requests at most twice a resource:

  $$\forall x, \mathtt{P_s}(x) \rightarrow \left[\forall y_1, y_2, y_3, \bigwedge_i \left(x \sim y_i \wedge \mathtt{req}(y_i)\right) \rightarrow \bigvee_{i \neq j} y_i = y_j\right]$$

Agent control

We consider three configurations:

1. All the agents belong to System



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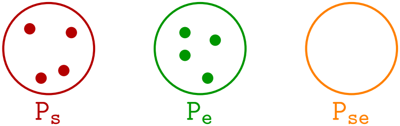
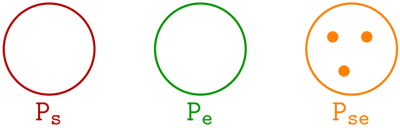


Agent control

We consider three configurations:

1. All the agents belong to System
2. There is no shared agent
3. All the agents are shared by System and Environment



Synthesis problem

Parameters:

- a logic (specification language) $\mathcal{L}$
- a configuration for agent control (System only, partitioned or shared)

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Synthesis problem for $\mathcal{L}$ for this configuration:

Input: a formula $\varphi \in \mathcal{L}$

Question: does there exist a distribution of agents, complying with the configuration, such that System has a winning strategy for φ?

Filling the gaps

Logic\Agents	System only[a]	Partitioned	Shared
$FO^2[\sim]$	decidable[1]	?	?
$FO[\sim]$	decidable[2]	?	undecidable[2]
$FO^2[\sim,<]$	decidable[1]	?	?
$FO^2[\sim,+1]$	decidable[1]	?	?
$FO^2[\sim,<,+1]$	decidable[1]	?	undecidable[2]

1: [Bojańczyk et al. '06]
2: [Bérard et al. '20]

[a] this amounts to the satisfiability problem

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$FO^2[\sim, +1]$	decidable[1]	**undecidable**	?
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Synthesis problem for $\mathrm{FO}^2[\sim, <]$ with partitioned agents

Two-counter Minksy machine:

- a finite set of states $\mathcal{Q}$ with $q_0, q_h \in \mathcal{Q}$

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  - decreasing a counter
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Run: sequence of states linked by transitions that do not

- decrease a counter below zero
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Halting run: run starting in q_0 with zero counters, and ending in q_h

Synthesis problem for $\mathsf{FO}^2[\sim, <]$ with partitioned agents

Halting problem for two-counter Minsky machines:

Input: a two-counter Minsky machine M

Question: does M have a halting run?

This problem is **undecidable**: we reduce it to the Synthesis problem for $\mathsf{FO}^2[\sim, <]$ with partitioned agents

Synthesis problem for $\mathrm{FO}^2[\sim, <]$ with partitioned agents

$\mathcal{Q} := \{q_0, q_1, q_2, q_h\}$ and $\mathcal{T} := \{t_0, t_1, t_2, t_3\}$, where $\begin{cases} t_0 : q_0 \xrightarrow{c_0++} q_0 \\ t_1 : q_0 \xrightarrow{c_0--} q_1 \\ t_2 : q_1 \xrightarrow{c_0--} q_2 \\ t_3 : q_2 \xrightarrow{c_0==0} q_h \end{cases}$

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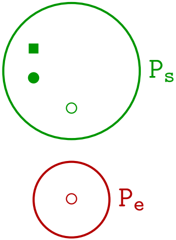
q_0 $\qquad$ $c_0 : 0$ $\qquad$ $c_1 : 0$

$(\circ, \mathrm{ok}_S)(\circ, \mathrm{ok}_E)(\circ, q_0)$



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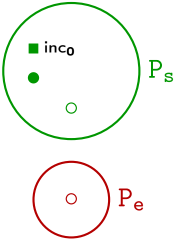
$q_0 \xrightarrow{t_0} q_0$ $\qquad$ $c_0 : 1$ $\qquad$ $c_1 : 0$

$(\circ, \mathrm{ok}_S)(\circ, \mathrm{ok}_E)(\circ, q_0)(\circ, t_0)(\blacksquare, \mathrm{inc}_0)(\circ, \mathrm{ok}_S)(\circ, \mathrm{ok}_E)$

$(\circ, q_0)$



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$q_0 \xrightarrow{t_0} q_0 \xrightarrow{t_0} q_0$ $\qquad c_0 : 2 \qquad c_1 : 0$

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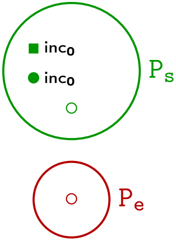

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$q_0 \xrightarrow{t_0} q_0 \xrightarrow{t_0} q_0 \xrightarrow{t_1} q_1$ $\quad c_0 : 1 \quad c_1 : 0$

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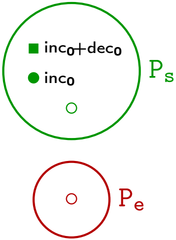

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$q_0 \xrightarrow{t_0} q_0 \xrightarrow{t_0} q_0 \xrightarrow{t_1} q_1 \xrightarrow{t_2} q_2$ $\qquad c_0 : 0 \qquad c_1 : 0$

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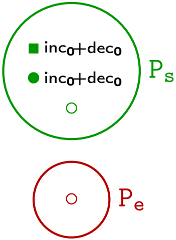

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$$q_0 \xrightarrow{t_0} q_0 \xrightarrow{t_0} q_0 \xrightarrow{t_1} q_1 \xrightarrow{t_2} q_2 \xrightarrow{t_3} q_h \qquad c_0 : 0 \qquad c_1 : 0$$

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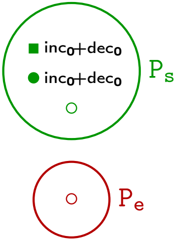
$(\circ, q_2)(\circ, t_3)(\circ, \mathrm{noop})(\circ, \mathrm{ok}_S)(\circ, \mathrm{ok}_E)(\circ, q_h)$



Conclusion

Logic\Agents	System only	Partitioned	Shared
$FO^2[\sim]$	decidable	**decidable**	**undecidable**
$FO[\sim]$	decidable	**decidable**	undecidable
$FO^2[\sim, <]$	decidable	**undecidable**	**undecidable**
$FO^2[\sim, +1]$	decidable	**undecidable**	**undecidable**
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Consider the intersection $\lesssim$ of $<$ and $\sim$

What about $FO^2[\lesssim]$ when agents are partitioned?