SUCCESSOR-INVARIANT FIRST-ORDER LOGIC ON CLASSES OF BOUNDED DEGREE

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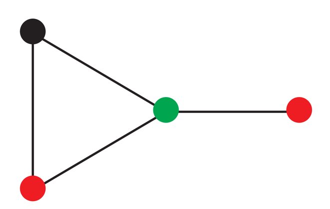
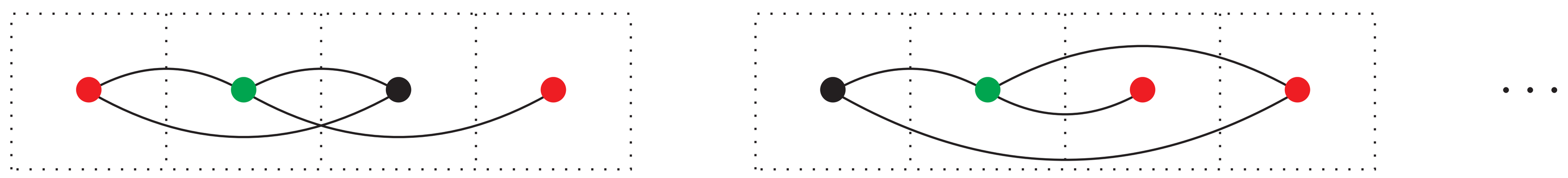
SUCCESSOR-INVARIANCE

A single abstract graph



has different representations in memory:



Vertices of the graph are ordered via a successor relation S.
Undesirable first-order sentence:

$$\varphi \quad := \quad \forall x, y, \quad \min(x) \wedge \max(y) \quad \rightarrow \quad E(x, y)$$

The truth value of φ in a graph depends on the representation $\rightsquigarrow$ it's not a **graph property**.

Definition. Succ-inv FO *is the set of* FO*-sentences using S, whose truth value is independent of the successor.*

Just like programs don't depend on the adjacency matrix, but only on the underlying graph.

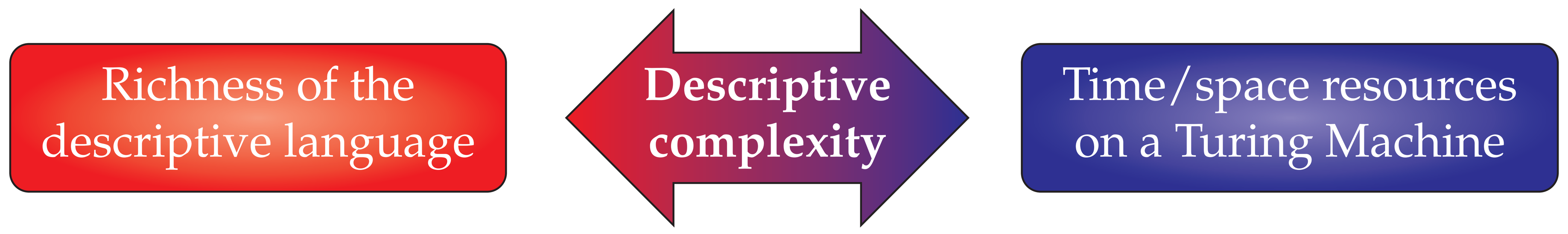
EXPRESSIVE POWER

What kind of graph properties are definable in Succ-inv FO?

- FO $\subseteq$ Succ-inv FO
- Succ-inv FO $\not\subseteq$ FO [Ros03]
- Succ-inv FO can only express *local properties* (e.g. `Reach` $\notin$ Succ-inv FO) [GS00]
- Succ-inv FO = FO on trees [BS05]

MOTIVATION

Successor-invariance is a central notion in **database theory** and **descriptive complexity**.



Succ-inv PFP		PSPACE
Succ-inv LFP	capture	PTIME
Succ-inv TC		NLOGSPACE
Succ-inv DTC		LOGSPACE

MAIN RESULT

Theorem. *When the degree is bounded,*

$$\text{Succ-inv FO} = \text{FO}\,.$$

In other words: when the degree is bounded, a property FO-definable (invariantly) using S is already definable without S.

FOLLOW-UP QUESTIONS

What happens...

- ...on other sparse classes of graphs?
- ...if we replace S with the full linear order?

PROOF TECHNIQUE

Starting point :

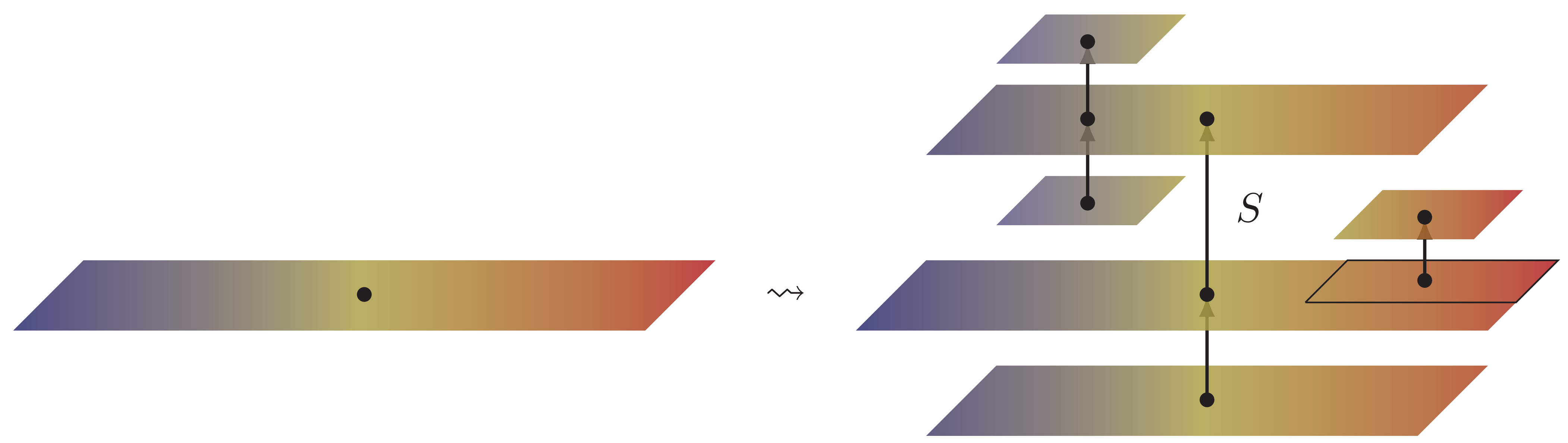
- $\mathcal{G}$ and $\mathcal{G}'$ of degree $\leq d$ in which occur the **same number of neighborhood types** up to some threshold.

Goal :

- construct successor relations S and S', s.t.
- $(\mathcal{G}, S)$ and $(\mathcal{G}', S')$ have the **same number of neighborhood types** up to some threshold.

This is enough, because FO is **local**.
The successors S and S' are constructed so that almost all neighborhoods in $(\mathcal{G}, S)$ and $(\mathcal{G}', S')$ are **fractal**.

FRACTAL NEIGHBORHOOD



Neighborhood of a in $\mathcal{G}$ Neighborhood of a in $(\mathcal{G}, S)$

Properties:

- v and $S(v)$ have the same neighborhood type in $\mathcal{G}$.
- No overlap between the neighrboods of $\mathcal{G}$ appearing in the fractal neighborhood.

This is the **most regular** way to extend a neighborhood of $\mathcal{G}$ with a successor relation.

REFERENCES

[BS05] Michael Benedikt and Luc Segoufin. Towards a characterization of order-invariant queries over tame structures. In *CSL*, 2005.

[GS00] Martin Grohe and Thomas Schwentick. Locality of order-invariant first-order formulas. *ACM Trans. Comput. Log.*, 2000.

[Ros03] Benjamin Rossman. Successor-invariance in the finite. In *LICS*, 2003.