Logic & Foundations
A personal perspective

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Typically one studies reasoning correct in some objective sense, not demagoguery.
My view of logic is expansive. Analogy:

homo economicus to real people,
homo logicus to real people.

Study argumentation for all audiences

- mathematicians, physicists, jury, family, government, humanity

and of any kind

- legal reasoning, political propaganda, and even demagoguery.

Again, my view is expansive.
Foundations for any science should be developed.

The topic of foundations richly deserves

- a broad sweep of the glorious past,
- examining the less glorious present, and
- discussing the uncertain future.

But time is short and my scholarship is limited. Hence:

1. a personal perspective and
2. a few illustrations instead of systematic exploration.

A discussion on the role of foundations.

If you find the talk is provocative, you are not wrong.
It is intended to be so.

When the roots are deep,
there is no reason to fear the wind.

Folk wisdom

There does not exist a category of science
to which one can give the name applied science.
There are sciences and the application of science,
bound together as a tree and the fruit it bears.

Louis Pasteur

It's not enough to know your worth;
you still need to be in demand.

Michael Zhvanetsky

(Мало знать себе цену;
надо ещё пользоваться спросом.)

- First exposure: Congruent triangles
- Ural State University and Novosibirsk Algebra and Logic seminar
- Jerusalem logic seminar
- University of Michigan

  What's an algorithm?
- Microsoft smorgasbord

  Software specification, verification, testing
  Access control, security, cybersecurity

  ⋮

  Quantum computing

A system of symbols for the basic sounds of the language.

People make a continuum of sounds varying between languages, dialects, and speakers of the same dialect.

The first alphabet: The continuum of consonant sounds (vowels play an auxiliary role in Semitic languages) was reduced to just a few, twenty two to be exact.

It seems miraculous that the alphabetic principle was conceived, was implemented, and was accepted.

It is an astounding idea; a triumph of syntax over semantics. Making alphabet sacred was probably instrumental.

Phoenician	Paleo-Hebrew	Hebrew letter (Dfus)	English name
𐤀	𐤀	א	Aleph
𐤁	𐤁	ב	Bet
𐤂	𐤂	ג	Gimel
𐤃	𐤃	ד	Dalet
𐤄	𐤄	ה	He
𐤅	𐤅	ו	Waw
𐤆	𐤆	ז	Zayin
𐤇	𐤇	ח	Heth
𐤈	𐤈	ט	Teth
𐤉	𐤉	י	Yodh
𐤊	𐤊	כ/ך	Kaph
𐤋	𐤋	ל	Lamedh
𐤌	𐤌	מ/ם	Mem
𐤍	𐤍	נ/ן	Nun
𐤎	𐤎	ס	Samekh
𐤏	𐤏	ע	Ayin
𐤐	𐤐	פ/ף	Pe
𐤑	𐤑	צ/ץ	Tsade
𐤒	𐤒	ק	Qoph
𐤓	𐤓	ר	Resh
𐤔	𐤔	ש	Shin
𐤕	𐤕	ת	Taw

	Proto-Canaanite	Early Phoenician	Greek		Proto-Canaanite	Early Phoenician	Greek
ʾ			A	l			Λ
b			B	m			M
g			Γ	n			N
d			Δ	s			Ξ
h			E	ʿ			O
w			Y	p			Π
z			Z	ṣ			M
ḥ			H	q			
ṭ			Θ	r			P
y			I	š			Σ
k			K	t			T

> "From the alphabet's earliest manifestation 4000 years ago, all other alphabets take their cue; and all reflect the idea's underlying simplicity."

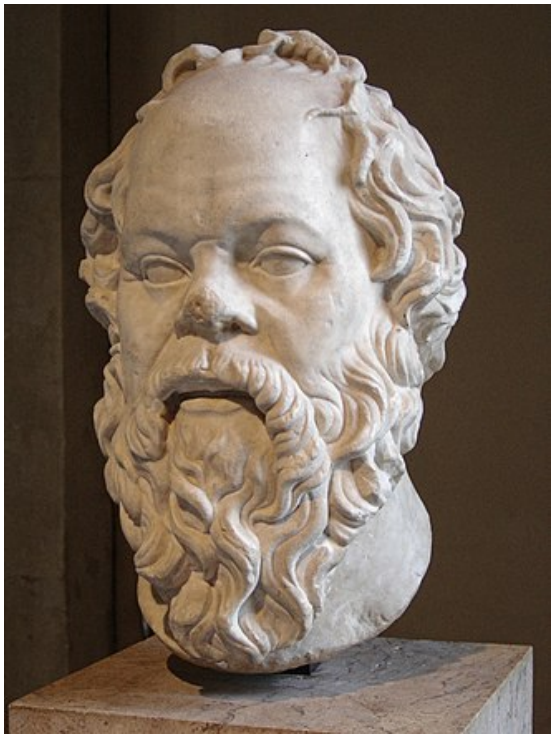
The secret behind the practicality of alphabet is ...
its imperfection.

> "This is not the simplicity of perfect design. The strength of the alphabet as an idea lies in its practical imperfection. Though it fits no language to perfection, it can, with some pushing and shoving, be adapted to all languages."

(The quotes are from "Alpha Beta" by John Man)

Plato's Theaetetus
($\Theta\varepsilon\alpha\iota\tau\eta\tau o\varsigma$) $\approx$ 369 BCE

D1: Knowledge is perception
D2: Knowledge is true belief
D3: Knowledge is true belief with an account



A marble head of Socrates
in the Louvre (from Wikipedia)

In 1591 “Introduction to the analytic arts” Franciscus Vieta used vowels for unknowns and consonants for constants.

François Viète (1540-1603)
(from Wikipedia)

We start this story with the discovery that the hypotenuse of an isosceles right triangle is incommensurable with its legs (i.e. that $\sqrt{2}$ is irrational) which contradicted a Pythagorean dictum according to which a small indivisible unit should fit evenly into all three sides of the triangle. Apparently the discoverer was Hippasus of Metapontum who might have been killed by Pythagorean zealots.

The theory of infinitesimals gave a different view on the problem; an infinitesimal seems to fit evenly into every side of any triangle.

Isaac Newton 1643–1727

Gottfried Wilhelm Leibniz
1646–1716

But of course infinitesimals themselves are problematic.
Mature theory of calculus avoided infinitesimals.
That is, until Robinson's nonstandard analysis, which settled our problem.


Abraham Robinson 1918–1974

No infinitesimal x fits evenly into 1 and $\sqrt{2}$, but there are nonstandard integers I and J such that Ix and Jx are infinitely close to 1 and $\sqrt{2}$ respectively.

Here are some contributors to the foundations of mathematics:

Bernard Bolzano (1781-1848),
Nikolai Lobachevsky (1792-1856),
Lejeune Dirichlet (1805-1859),
George Boole (1815-1864),
Karl Weierstrass (1815-1897),
Richard Dedekind (1831-1916),
Georg Cantor (1845-1918),
Gottlob Frege (1848-1925),
Giuseppe Peano (1858-1932),
David Hilbert (1862-1943),
Ernst Zermelo (1871-1953),
Bertrand Russell (1872-1970),
Leopold Löwenheim (1878-1957),
Hermann Weyl (1885-1955),
Thoralf Skolem (1887-1963),
Abraham Fraenkel (1891-1956),
Emil Post (1897-1954),
Alfred Tarski (1901-1983),
Andrei Kolmogorov (1903-1987),
John von Neumann (1903-1957),
Kurt Gödel (1905-1978),
Alan Turing (1912-1954)

Constructivist

1918 "The continuum," most of classical calculus, predicatively, without proof by contradiction, infinite sets; a bet with George Pólya

1921 "On the new foundational crisis of mathematics," admitted numbers in a perpetual state of becoming, "that is the revolution!"

Pragmatic scientist

Some books: 1918 "Space, time, matter," 1926 "Theory of groups and quantum mechanics," 1939 "The classical groups," 1949 "Philosophy of mathematics and natural sciences," 1952 "Symmetry"

As we grow older
The world
becomes stranger,
the pattern
more complicated

T.S. Eliot

Collected works, Pergamon Press, 1961-1963

- Vol 1 Logic, theory of sets and quantum mechanics
- Vol 2 Operators, ergodic theory and almost periodic functions on a group
- Vol 3 Ring of operators
- Vol 4 Continuous geometry and other topics
- Vol 5 Design of computers, theory of automata and numerical analysis
- Vol 6 Theory of games, astrophysics, hydrodynamics and meteorology

1930

For a while after WW2, logic was rather popular and foundational.

- The first computers were designed on a logic foundation.
- Symbolic artificial intelligence dominated for quite a while other AI approaches like statistical and cybernetics.
- Logic programming was quite a fad.
- Non-standard analysis.
- Forcing revolutionized set theory.

But then things started to change.

While the need for logic foundation research never was greater, less and less attention is given to foundational issues, even in areas like set theory.

By and large, logic factions are slowly fizzling out at top Mathematics and Philosophy departments.
For example, the Math Dept of ETH Zurich goes from having Zermelo, Bernays, Specker, etc. to no logicians.

There are many logicians in computer science but they don't do much foundational work there. In particular, the golden age of logic in AI is behind us. The role of logic is taken by statistics.

Scientific progress will continue. Foundational problems will inevitably arise and will be addressed. But what role will logicians play?

The future may be bleak. Foundational logic research fades away. Significant logic areas become parts of mathematics or computer science.

As T.S. Eliot said in "The Hollow Men,"

Between the potency
And the existence
Between the essence
And the descent
Falls the Shadow
⋮
This is the way the world ends
Not with a bang but a whimper.

But this does not have to be so. The future could be bright, albeit challenging. First of all, I think, we need to discuss the issue explicitly. Logic research should recover its foundational spirit.

- **More comprehensive logic research**

  To me, logic is a science of convincing arguments.
  It does not reduce to the deductive mathematical logic.
  Mathematics may be the last science that needs outside deduction experts because mathematicians themselves are experts in deduction.

- **More comprehensive foundation research**

  Throughout ages, logicians made great contributions to the foundations of various sciences including mathematics but also natural sciences.

There are many foundational problems to explore. In the rest of this section, I give a few rather general examples.

The list is not the result of a careful investigation. These are the examples that occurred to me as I was preparing the (original) talk.

Already Aristotle mentioned inductive inference in addition to deductive. Yet, inductive inference remains a challenge.

The problem has been addressed by philosophers, notably Hume and Popper. Popper's falsifiability principle has been influential with natural scientists. Critics of Popper point out that his principle is simplistic for natural sciences. Pragmatic natural scientists use Popper's principle as a general guide, not literally.

Bayesian inference is explicitly pragmatic. You don't aim to decide whether the hypothesis H in question is true or false. Instead you aim just to confirm or disconfirm H to the extent of the available evidence. Bayesian inference is a useful and widely used tool.

What is information? What is the algebra of information? What is the logic of information? My coauthors and I attempted to address the issue in article on infon logic.

When we worked on inverse privacy, it became clear that the logic foundations of privacy are all but nonexistent. Experts disagree even on what privacy is.
We started a foundational development but carried it out only to the small extent sufficient for our needs in that article.

The same questions arise for knowledge and for privacy instead of information. As far as knowledge is concerned, we have more data than Plato did but only limited progress so far.

Neuroscience is one of the most fascinating and fast developing sciences. Can foundational logic research be useful in neuroscience and life sciences in general? This is hard to tell. “If people do not believe that mathematics is simple,” wrote John von Neumann, “it is only because they do not realize how complicated life is."But we will not know the answer to the question above if we don't try.

A more specific question is related to Daniel Kahneman's *Thinking fast and slow*. The slow thinking is the conscious thinking of homo sapiens. The fast thinking is the unconscious thinking that homo sapiens inherited from preceding species. What is the logic of fast thinking? It is much different from Bayesian inference as Kahneman convincingly argues.

The advance in digital technology enabled social scientists to collect and analyze large data and made social sciences closer to traditional hard sciences like physics; see Duncan Watts's *Everything is obvious (once you know the answer)*.

Social sciences may be even more involved than life sciences. Can foundational logic research be useful there? I don't see why not. We certainly should try. For example, can one objectively define the degrees of spin and the fakeness of news?

Compared to legal reasoning, mathematical reasoning is easy. Outside mathematics, human pronouncements may not be — and usually are not — either true or false. They may be too vague, for example. Also, the truth value of the pronouncement may be largely irrelevant because the real purpose is to express an emotion.

> What is needed in law, if law is to become more scientific in the future . . . , is a body of learning from which we can predict that what looks like a straight story . . . will look like a crooked story . . . from another, and from which we can predict the successive "distortions". . .
>
> Felix Cohen, *Field theory and judicial logic*

Of course such predictions would be of various degrees of confidence. It would be useful to reason with predictions.

"The 'expansive' view that you take of the foundations of logic seems to me not far from the consensus up to the beginning of the nineteenth century.

For example al-Farabi classified kinds of logic by the practical effects that they were designed to achieve. One kind of logic would attain incontrovertible truth, another kind was for winning people over to your opinions. What look to us like his inference rules were for him the rules that society had come to agree were good for settling disagreements between two individuals.

I think he was being largely faithful to Aristotle with these views."

Thank you