Functional abstraction for programming Multi-level architectures:

Formalisation and implementation

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under the supervision of:
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Ph.D. defense

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2. The MULTI-ML language
3. Type system
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- The world of parallel computing
- Parallel programming models

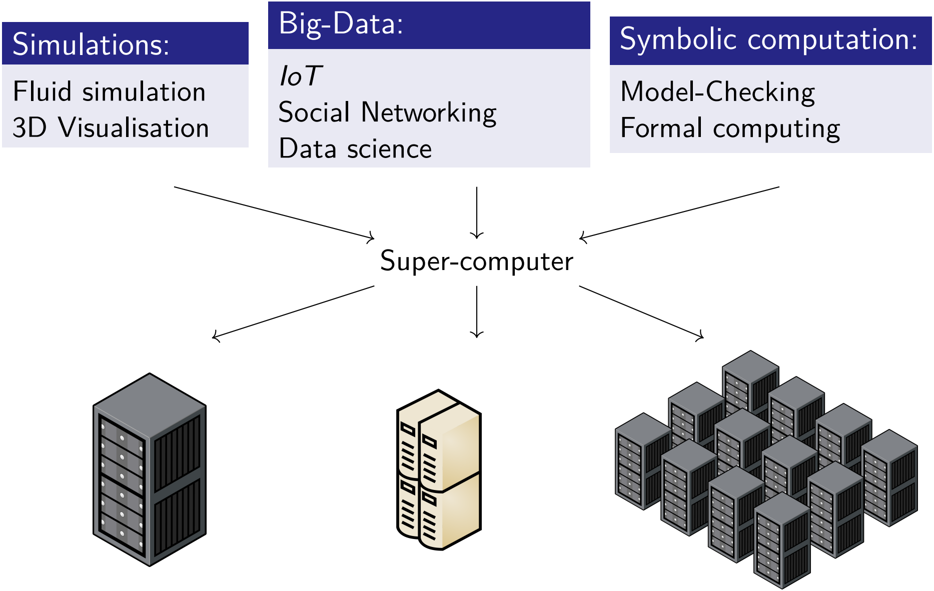
2 The MULTI-ML language

3 Type system

4 Implementation

5 Conclusion

The world of parallel computing



Simulations:

Fluid simulation
3D Visualisation

Big-Data:

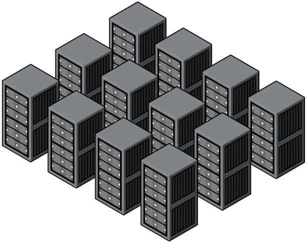
IoT
Social Networking
Data science

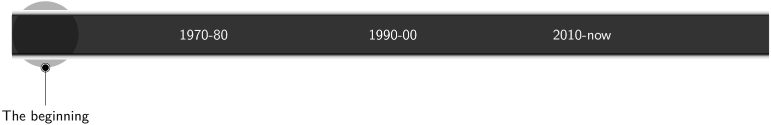
Symbolic computation:

Model-Checking
Formal computing

Super-computer

Parallel computing over the years





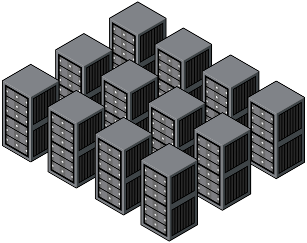
1970-80

1990-00

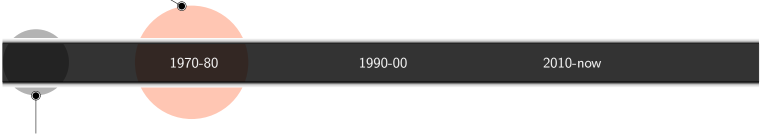
2010-now

The beginning

Parallel computing over the years

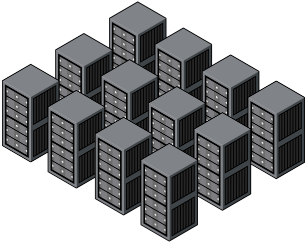


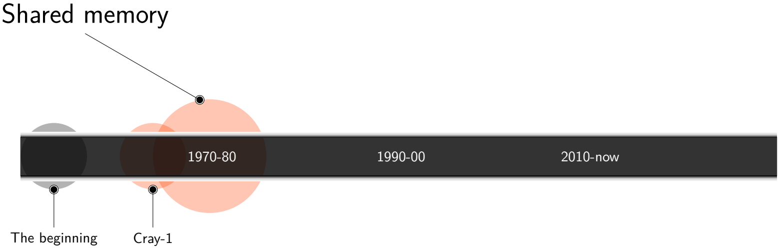
Shared memory



The beginning

Parallel computing over the years



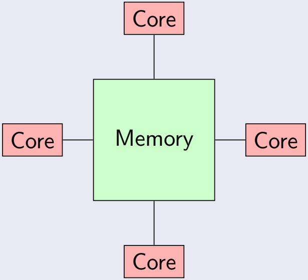


Shared memory

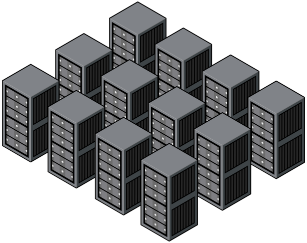
Shared memory models

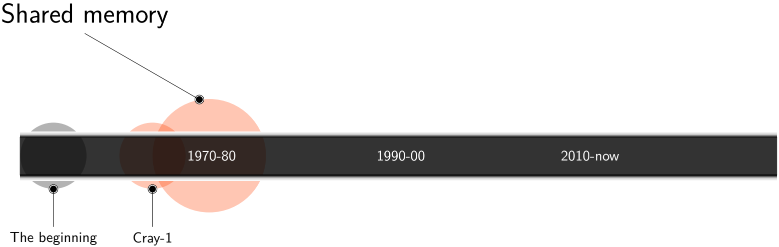
Characterised by:

- A shared memory
- Integrated network (NUMA)
- OPENMP/PTHREAD (C, FORTRAN)



Parallel computing over the years





Shared memory

1970-80

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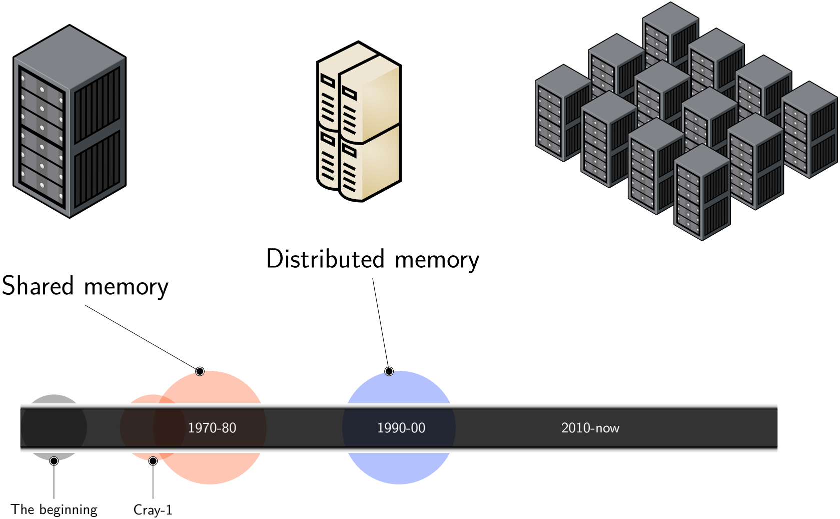
2010-now

The beginning

Cray-1

Parallel computing over the years



Distributed memory

Shared memory

1970-80

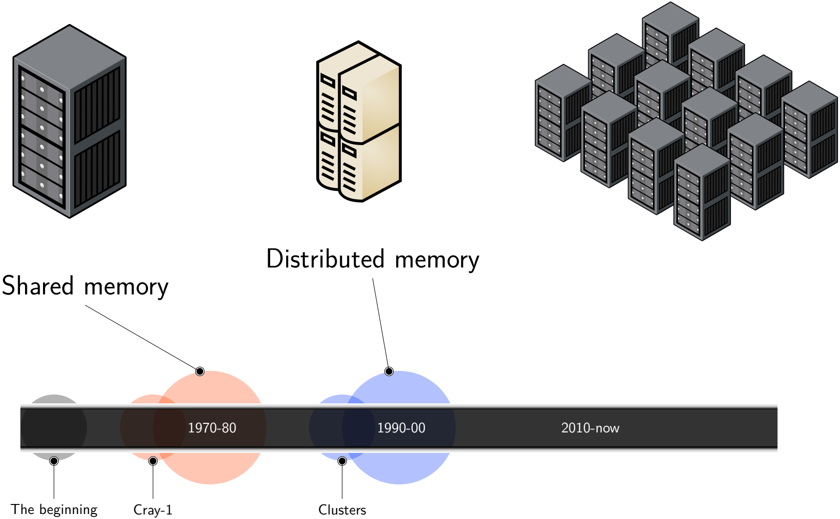
1990-00

2010-now

The beginning

Cray-1

Parallel computing over the years



Distributed memory

Shared memory

1970-80

1990-00

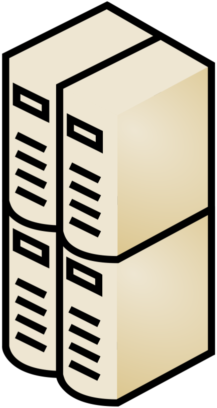
2010-now

The beginning

Cray-1

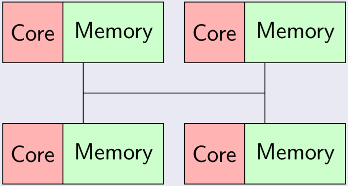
Clusters

Distributed computing

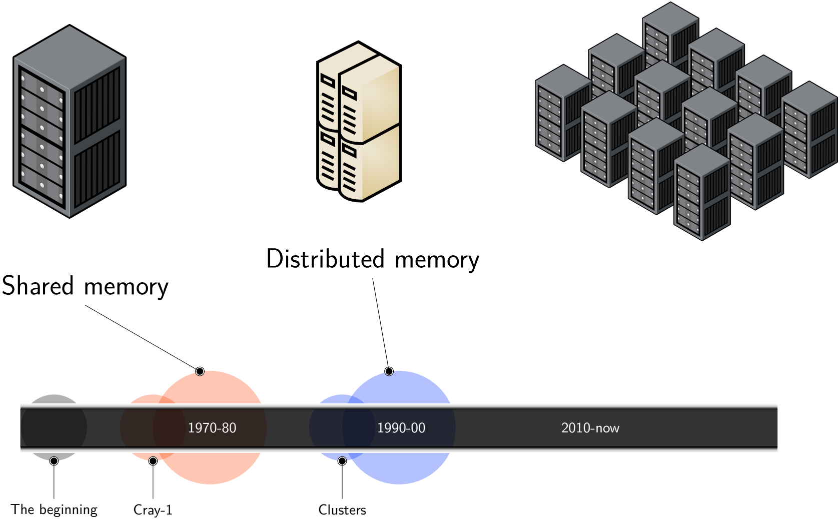


Characterised by:

- Interconnected units
- Distributed memory
- Communication network
- MPI/map-reduce



Parallel computing over the years



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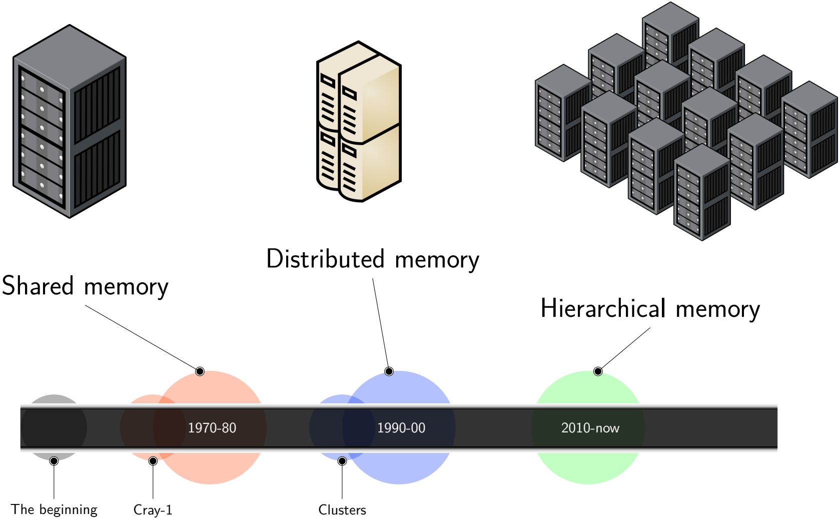

Shared memory

Distributed memory

Hierarchical memory

1970-80

1990-00

2010-now

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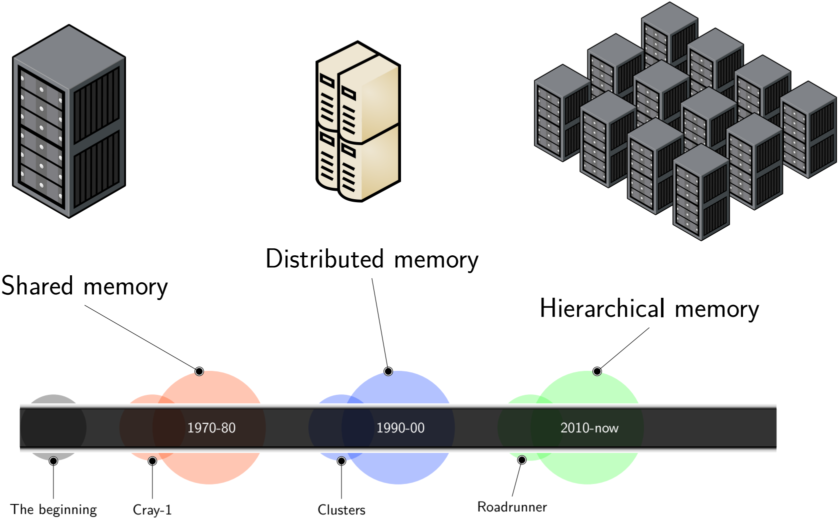
Clusters

Parallel computing over the years



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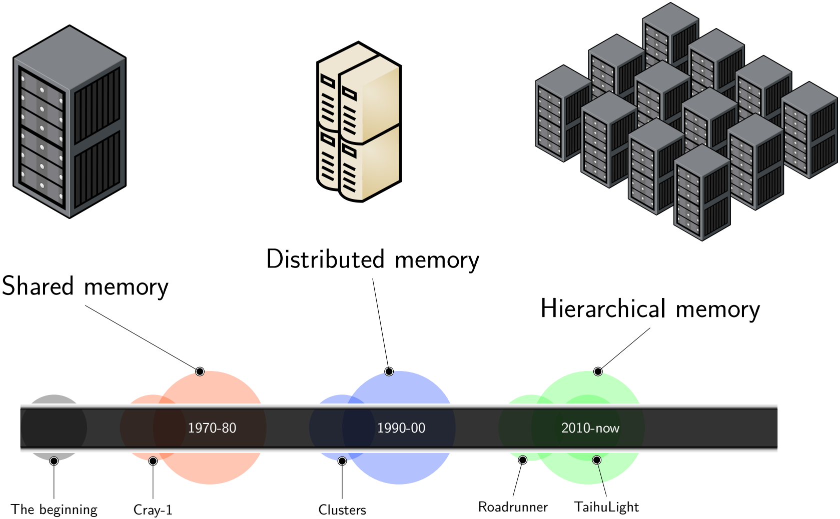
The beginning

Cray-1

Clusters

Roadrunner

Parallel computing over the years



Shared memory

Distributed memory

Hierarchical memory

1970-80

1990-00

2010-now

The beginning

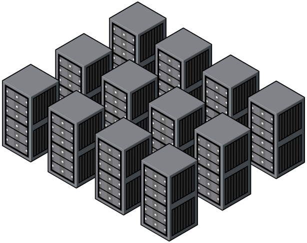
Cray-1

Clusters

Roadrunner

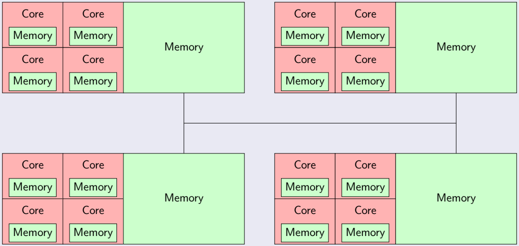
TaihuLight

Hierarchical architectures

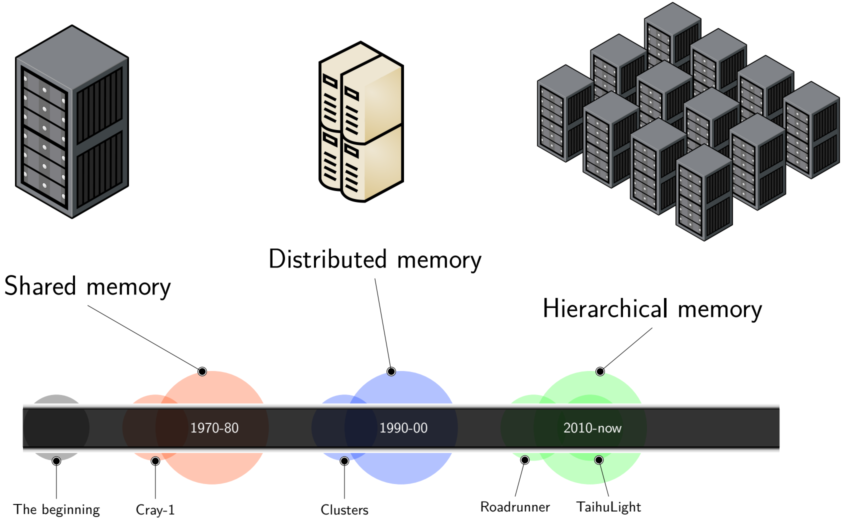


Characterised by:

- Interconnected units
- Both shared and distributed memories
- Hierarchical memories



Parallel computing over the years



Shared memory

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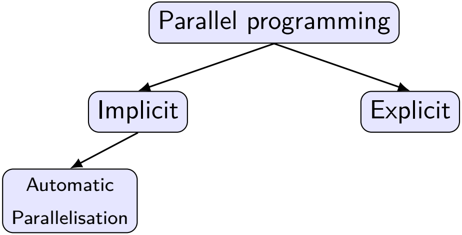
Parallel programming models



Automatic Parallelisation:

+ Easy

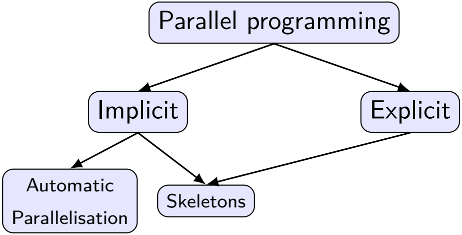
+ Transparent

− Limited

− “Naive”

- Par4All
- Intel C++ compiler
- Vienna Fortran compiler

Parallel programming models

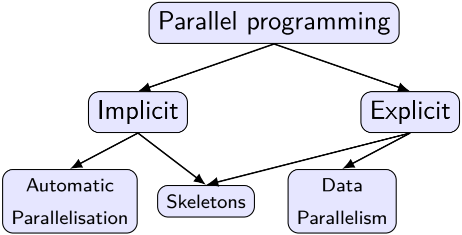


Skeletons:

+ Easy
+ Structured
– Difficult to extend
– Cost model

- SKML
- SKETO
- Muesli

Parallel programming models

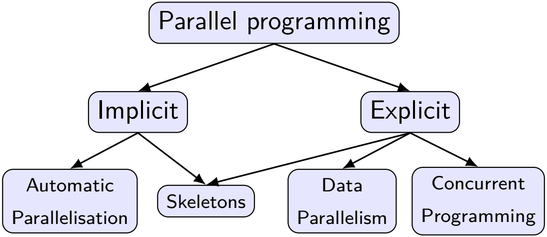


Data Parallelism:

+ Structured
+ Patterns
− Limited
− Complex

- OPENMP
- SAC
- CUDA

Parallel programming models

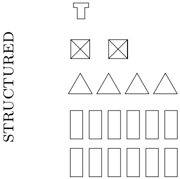


Concurrent Programming:

+ Flexible
+ Powerful
− Complex
− Error prone

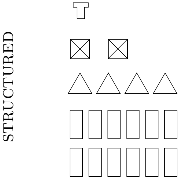
- MPI
- PTHREAD
- ERLANG/JOCAML

Why structured parallelism ?

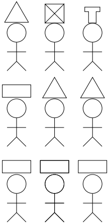


Pieces (Data) Workers (Processes) House (Results)

Why structured parallelism ?



+



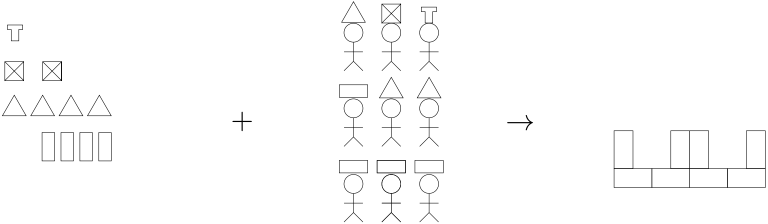
Pieces (Data)

Workers (Processes)

House (Results)

Why structured parallelism ?

STRUCTURED



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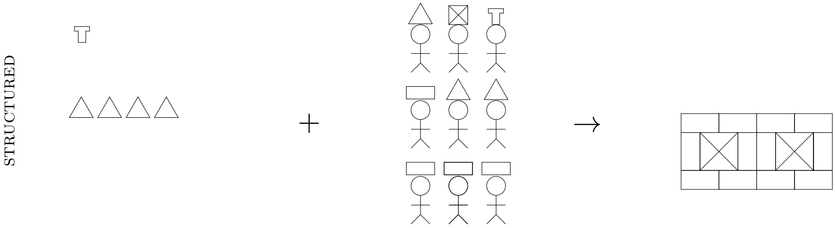

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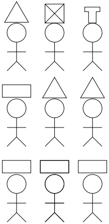

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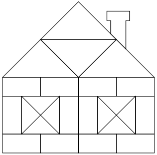

Pieces (Data) Workers (Processes) House (Results)

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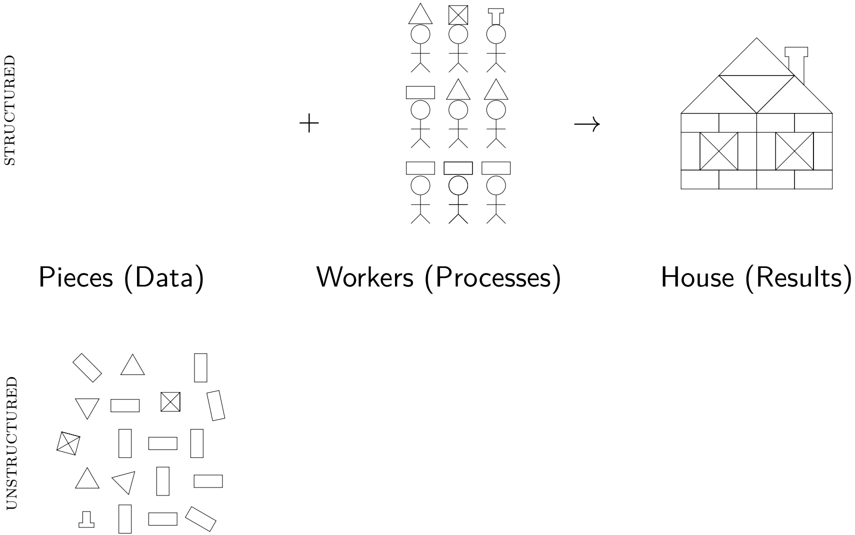


STRUCTURED

+

→

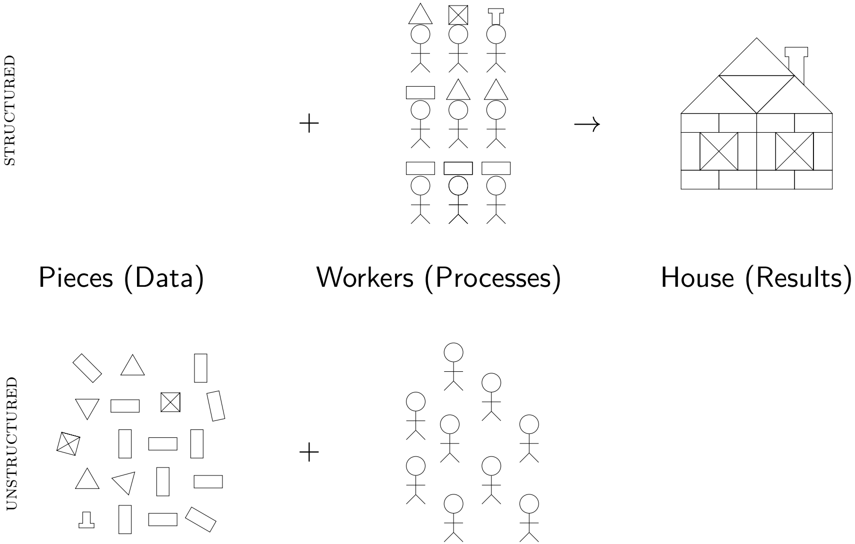
Pieces (Data)

Workers (Processes)

House (Results)

UNSTRUCTURED

Why structured parallelism ?



STRUCTURED

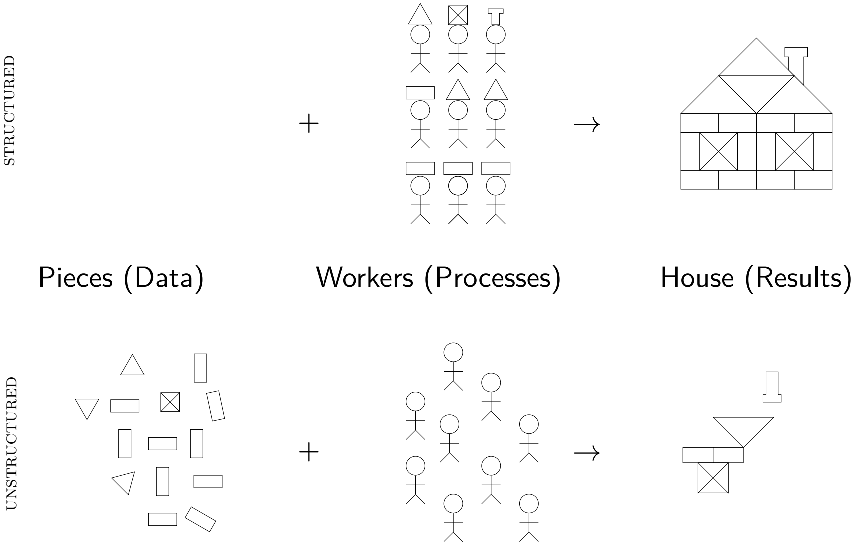
UNSTRUCTURED

Pieces (Data)

Workers (Processes)

House (Results)

Why structured parallelism ?



STRUCTURED

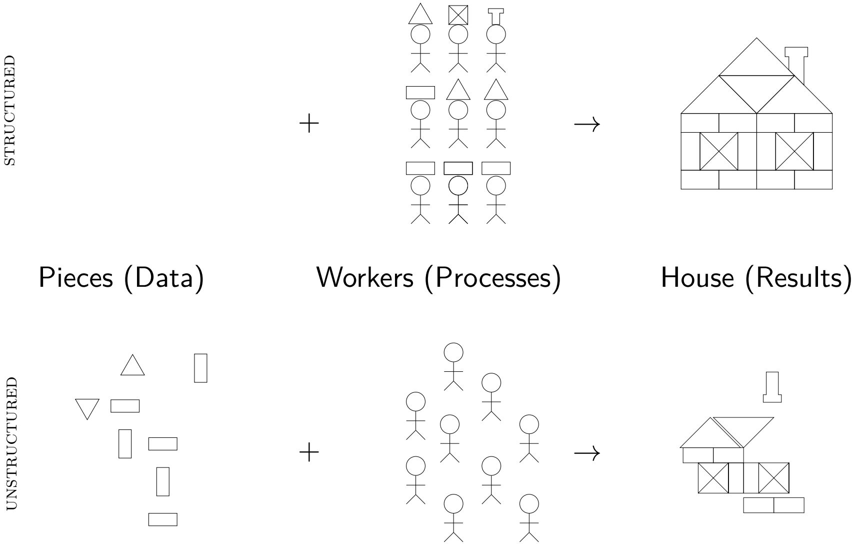
Pieces (Data)

Workers (Processes)

House (Results)

UNSTRUCTURED

Why structured parallelism ?



STRUCTURED

+ →

Pieces (Data)

Workers (Processes)

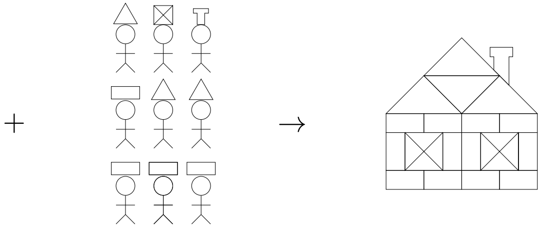
House (Results)

UNSTRUCTURED

+ →

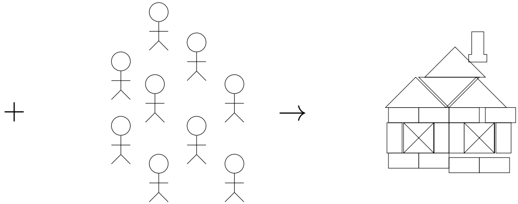
Why structured parallelism ?

STRUCTURED

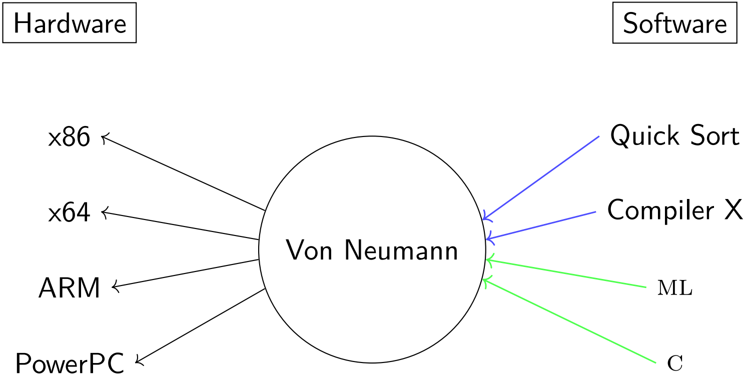


Pieces (Data) Workers (Processes) House (Results)

UNSTRUCTURED



A sequential bridging model



Hardware

Software

x86

x64

ARM

PowerPC

Von Neumann

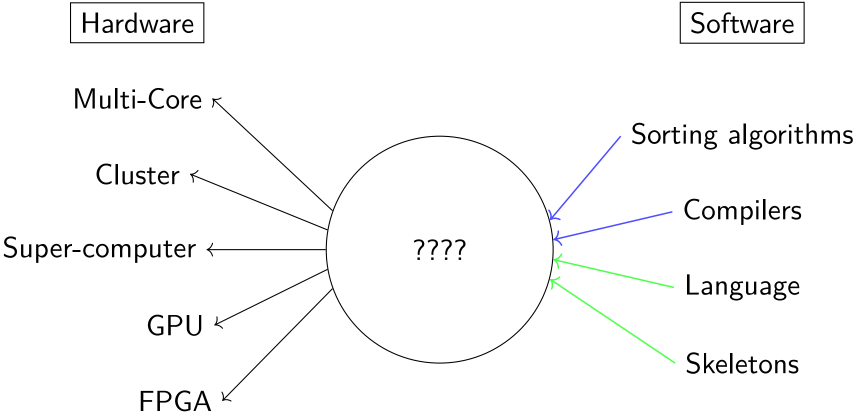
Quick Sort

Compiler X

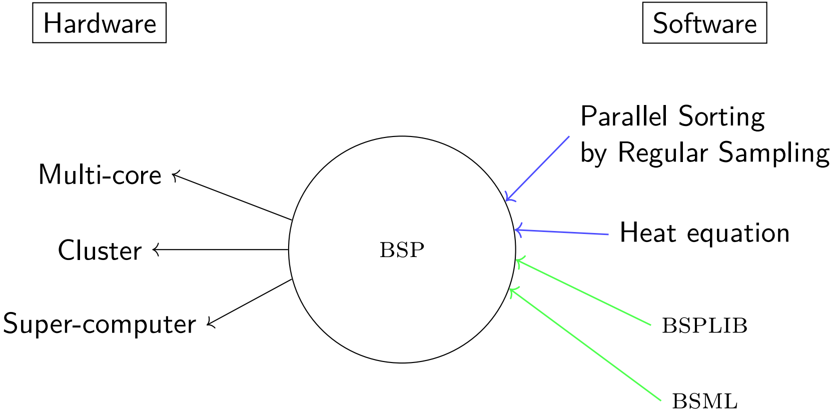
ML

C

A parallel bridging model



A parallel bridging model



Hardware

Software

Multi-core

Cluster

Super-computer

BSP

Parallel Sorting
by Regular Sampling

Heat equation

BSPLIB

BSML

Bulk Synchronous Parallelism

The BSP computer

Defined by:

Bulk Synchronous Parallelism

The BSP computer

Defined by:

- $\mathbf{p}$ pairs CPU/memory

Bulk Synchronous Parallelism

The BSP computer

Defined by:

- $\mathbf{p}$ pairs CPU/memory
- Communication network

Bulk **S**ynchronous **P**arallelism

The BSP computer

Defined by:

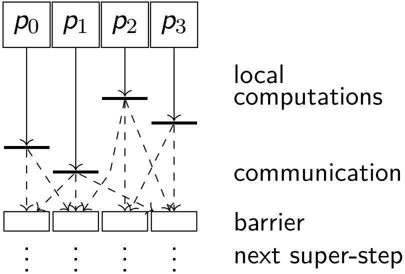
- $\mathbf{p}$ pairs CPU/memory
- Communication network
- Synchronisation unit

Bulk Synchronous Parallelism

The BSP computer

Defined by:

- **p** pairs CPU/memory
- Communication network
- Synchronisation unit
- Super-steps execution



Bulk Synchronous Parallelism

The BSP computer

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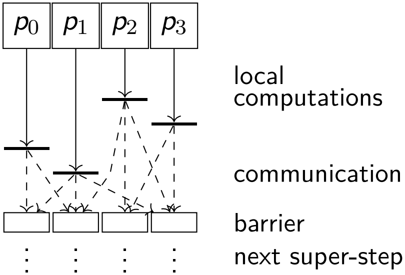
- Super-steps execution

Properties:



Bulk Synchronous Parallelism

The BSP computer

Defined by:

- **p** pairs CPU/memory
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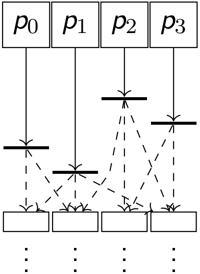
- Synchronisation unit
- Super-steps execution

Properties:

- Deadlock-free



local computations

communication

barrier

next super-step

Bulk Synchronous Parallelism

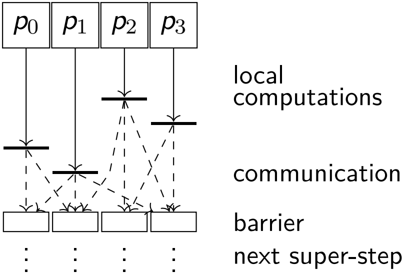
The BSP computer

Defined by:

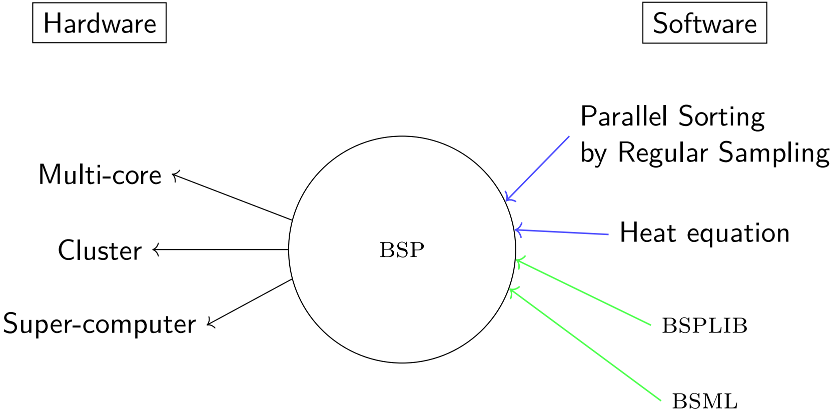
- **p** pairs CPU/memory
- Communication network
- Synchronisation unit
- Super-steps execution

Properties:

- Deadlock-free
- Predictable performances

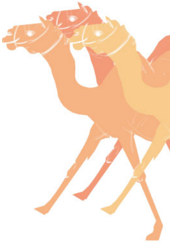


A parallel bridging model



What is BSML?

- Explicit BSP programming with a functional approach



Bulk Synchronous ML

What is BSML?

- Explicit BSP programming with a functional approach
- Based upon ML and implemented over OCAML

Bulk Synchronous ML

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- Formal semantics → computer-assisted proofs (COQ)

Bulk Synchronous ML

What is BSML?

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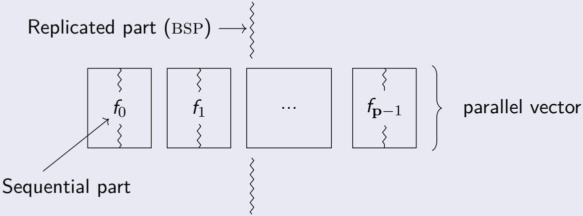
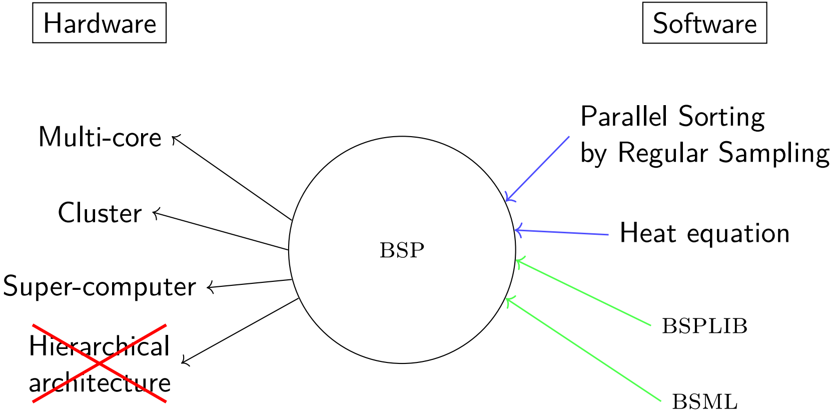
- Formal semantics → computer-assisted proofs (COQ)

Main idea

Parallel data structure ⇒ *parallel vector*:



A parallel bridging model



Hardware

Multi-core

Cluster

Super-computer

~~Hierarchical architecture~~

BSP

Software

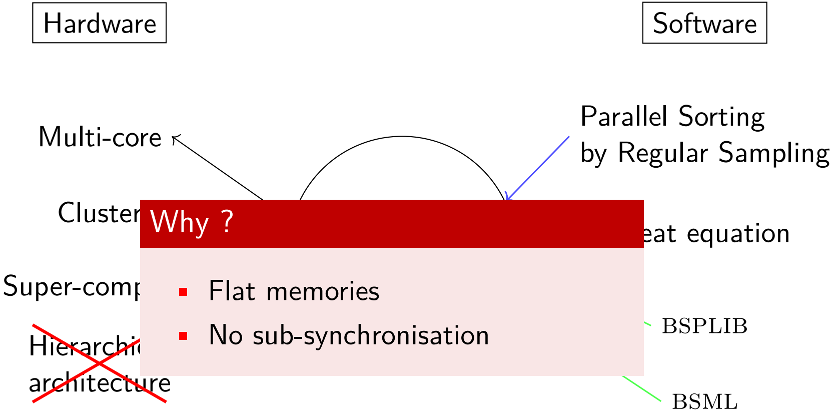
Parallel Sorting by Regular Sampling

Heat equation

BSPLIB

BSML

A parallel bridging model



Hardware

Software

Multi-core

Cluster

Super-comp

~~Hierarchic architecture~~

Parallel Sorting by Regular Sampling

eat equation

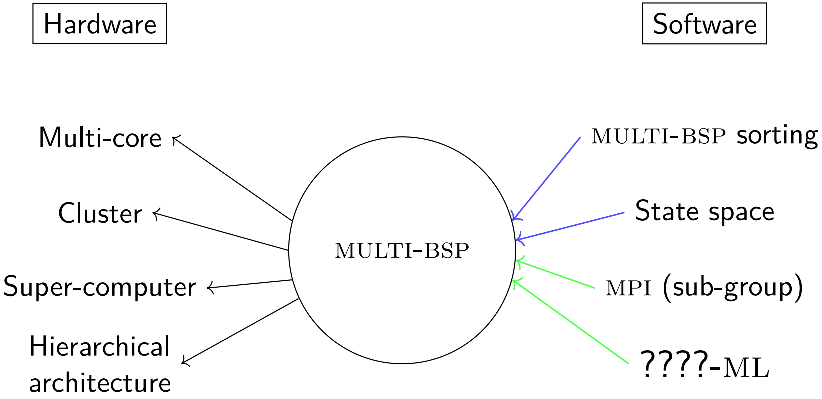
BSPLIB

BSML

Why ?

- Flat memories
- No sub-synchronisation

A parallel bridging model



Hardware

Software

Multi-core

Cluster

Super-computer

Hierarchical architecture

MULTI-BSP

MULTI-BSP sorting

State space

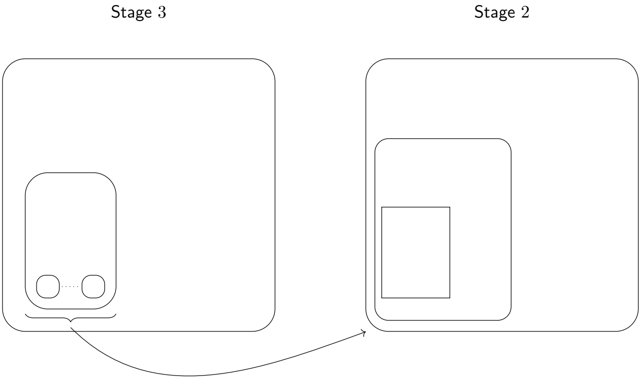
MPI (sub-group)

????-ML

What is MULTI-BSP?

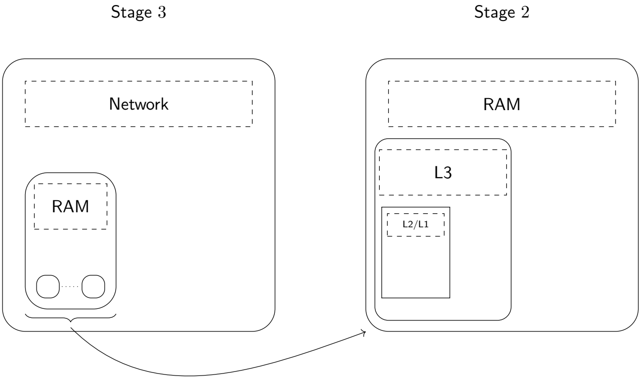
What is MULTI-BSP?

1. A tree structure with nested components



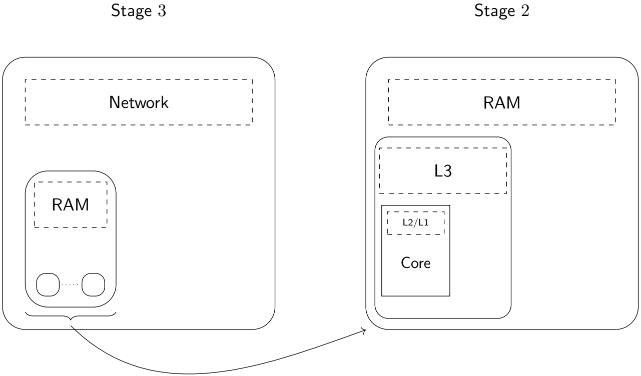
What is MULTI-BSP?

1. A tree structure with nested components
2. Where nodes have a storage capacity



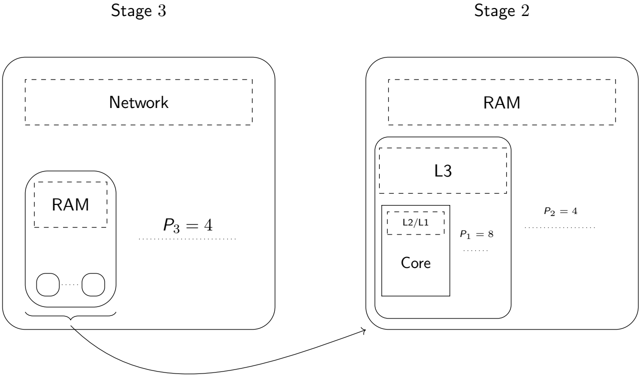
What is MULTI-BSP?

1. A tree structure with nested components
2. Where nodes have a storage capacity
3. And leaves are processors



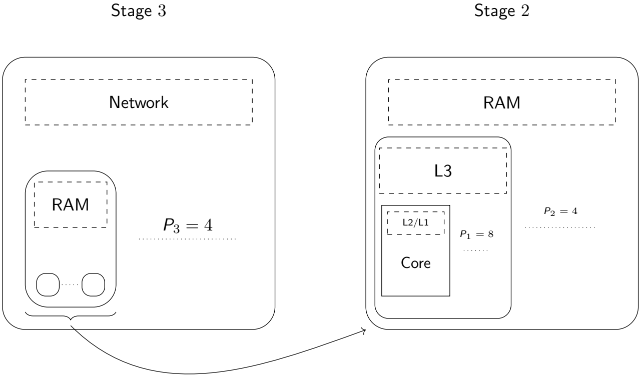
What is MULTI-BSP?

1. A tree structure with nested components
2. Where nodes have a storage capacity
3. And leaves are processors
4. With sub-synchronisation capabilities



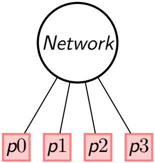
What is MULTI-BSP?

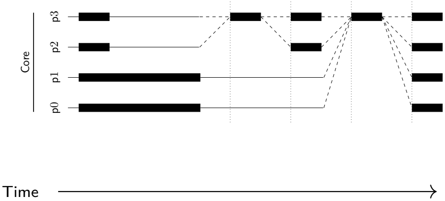
- Stage 3: 4 nodes with a network access
- Stage 2: one node has 4 chips plus RAM
- Stage 1: one chip has 8 cores plus L3 cache
- Stage 0: one core with L1/L2 caches



BSP vs. MULTI-BSP

BSP





BSP vs. MULTI-BSP

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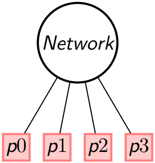
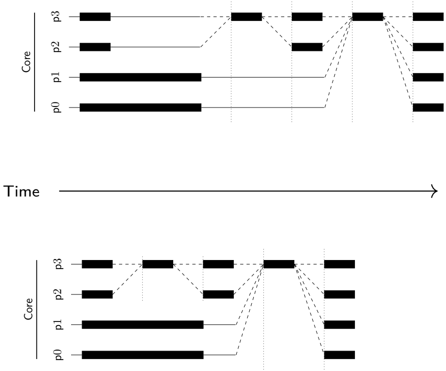
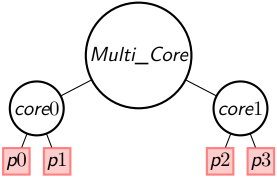




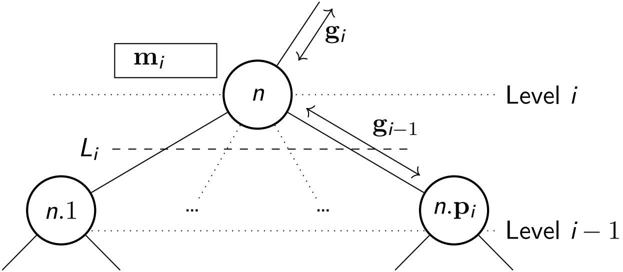
MULTI-BSP



The MULTI-BSP model

Execution model

A level *i* superstep is:

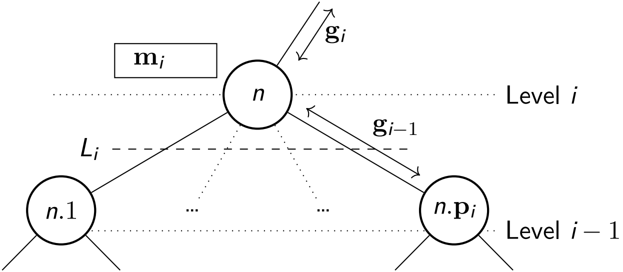


The MULTI-BSP model

Execution model

A level i superstep is:

- Level $i-1$ executes code independently

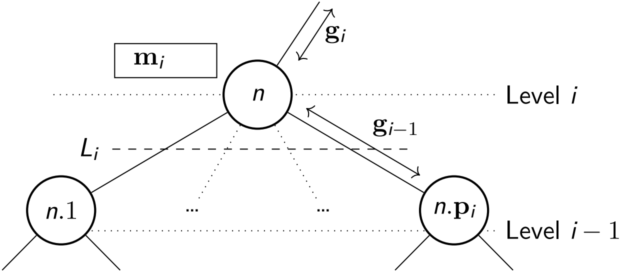


The MULTI-BSP model

Execution model

A level i superstep is:

- Level $i-1$ executes code independently
- Exchanges information with the m_i memory

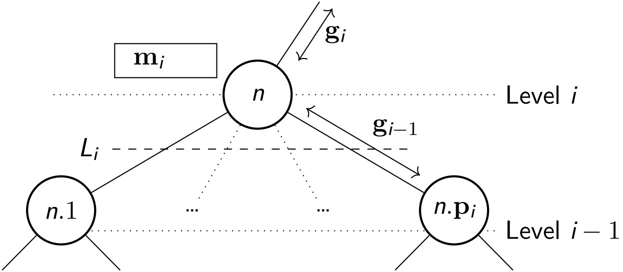


The MULTI-BSP model

Execution model

A level i superstep is:

- Level $i-1$ executes code independently
- Exchanges information with the m_i memory
- Synchronises



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1 Introduction

2 The MULTI-ML language

- MULTI-ML overview
- The MULTI-ML primitives
- A code example

3 Type system

4 Implementation

5 Conclusion

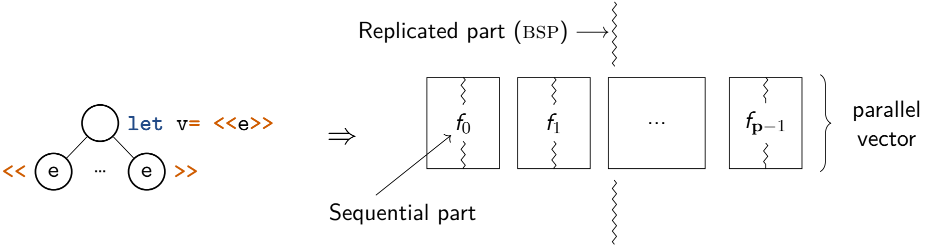
The MULTI-ML language

Basic ideas

The MULTI-ML language

Basic ideas

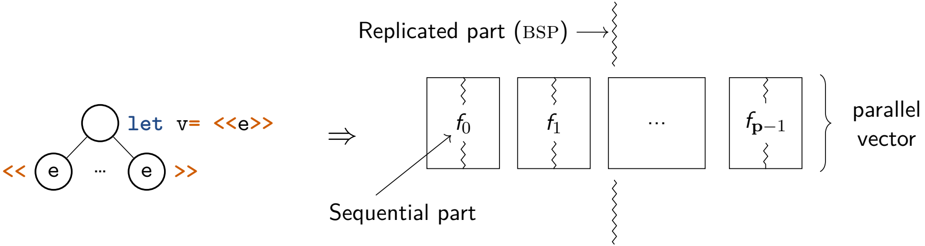
- BSML-like code on every stage of the MULTI-BSP architecture



The MULTI-ML language

Basic ideas

- BSML-like code on every stage of the MULTI-BSP architecture
- Specific syntax over ML: eases programming



```
let v= <<e>>
```

<< e ... e >>

⇒

Replicated part (BSP) ⟶

f_0 f_1 $\cdots$ $f_{\mathbf{p}-1}$

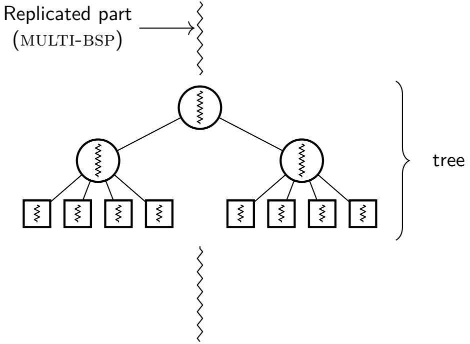
parallel vector

Sequential part

The MULTI-ML language

Basic ideas

- BSML-like code on every stage of the MULTI-BSP architecture
- Specific syntax over ML: eases programming
- *Multi-functions* that recursively go through the MULTI-BSP tree



MULTI-ML: Tree recursion

Recursion structure

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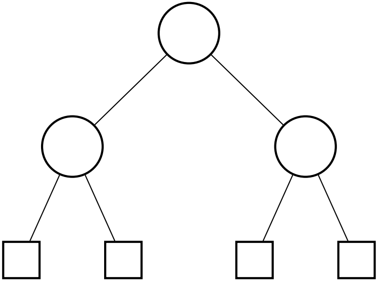

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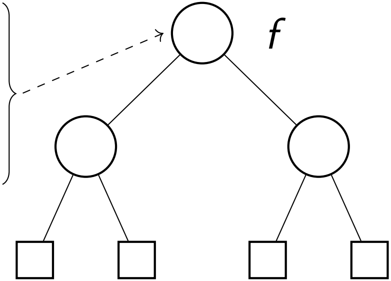

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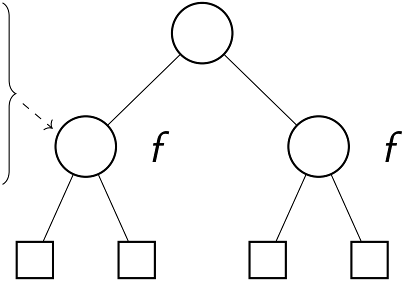

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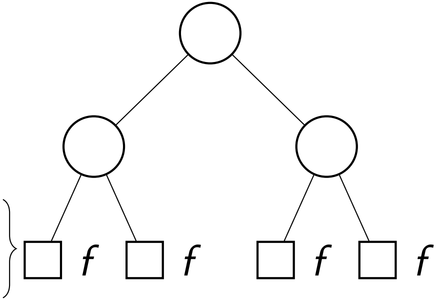

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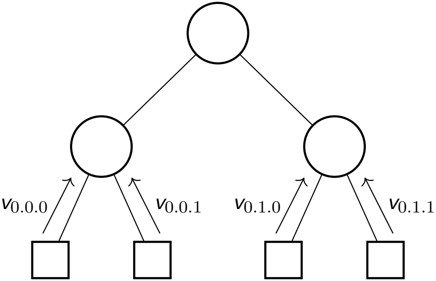

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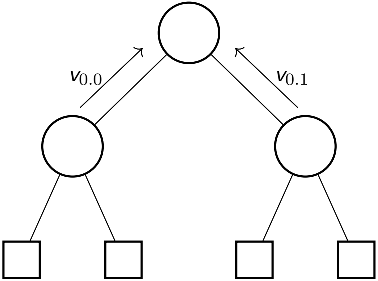

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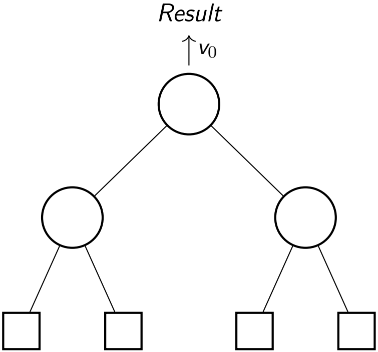

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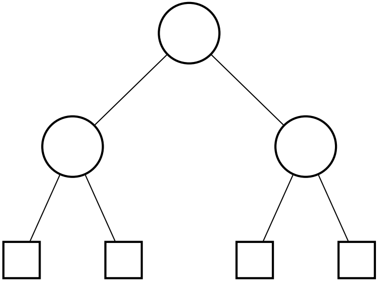

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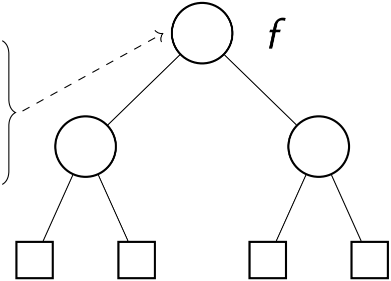

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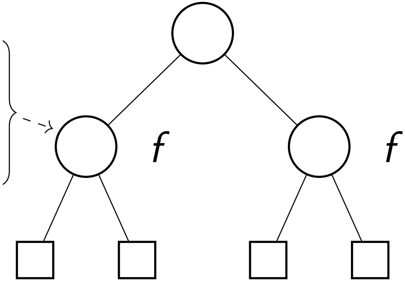

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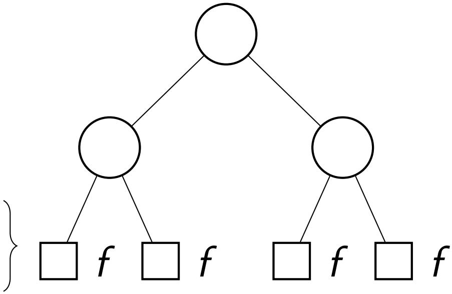

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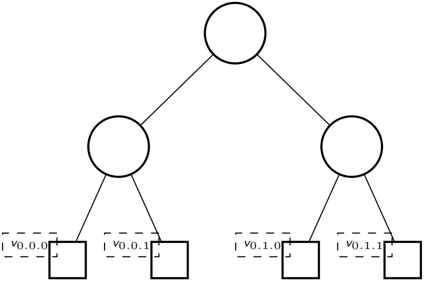

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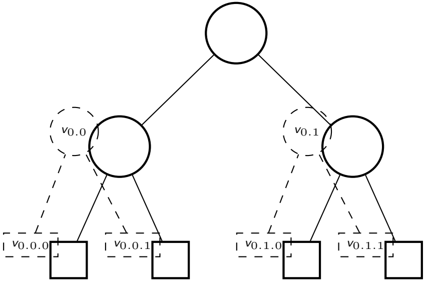

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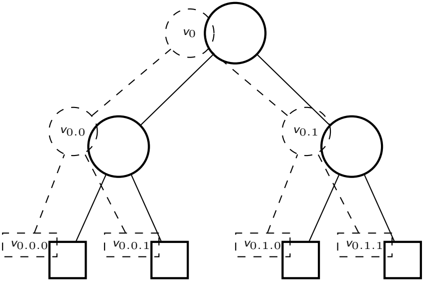

Primitives

Summary

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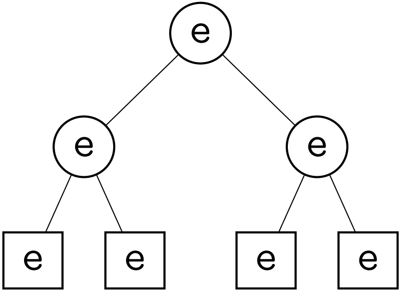
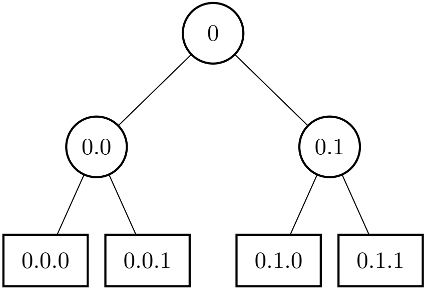
- `mktree e`



Primitives

Summary

- `mktree e`
- `gid`



Primitives

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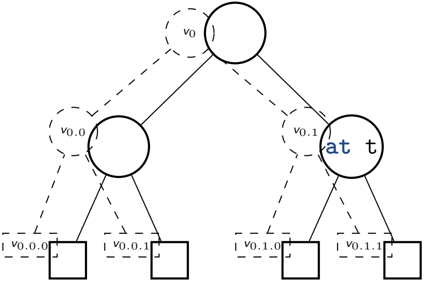

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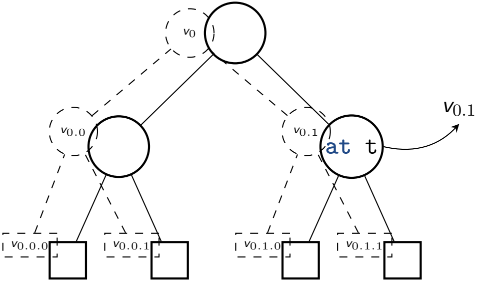

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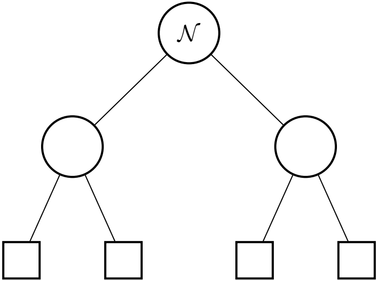

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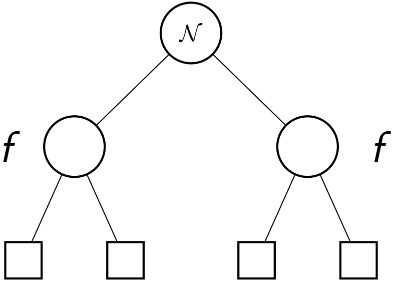

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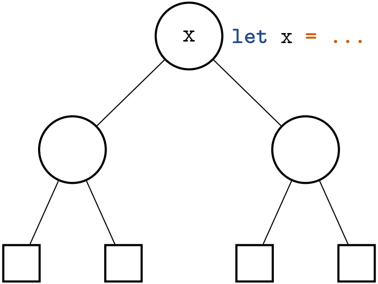

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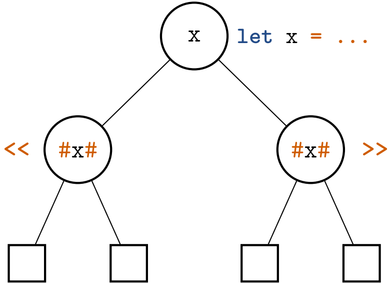

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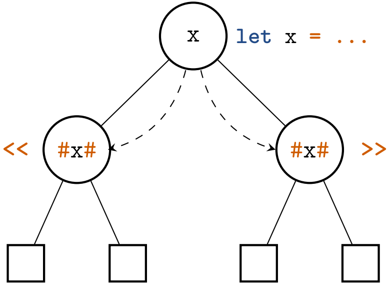

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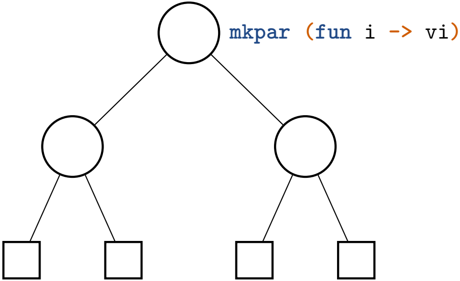

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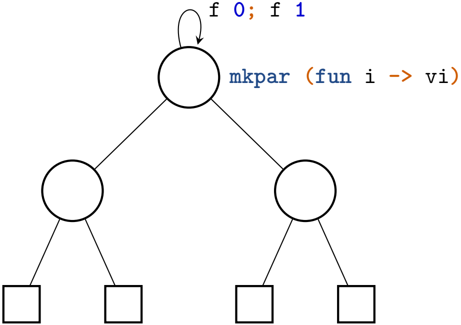

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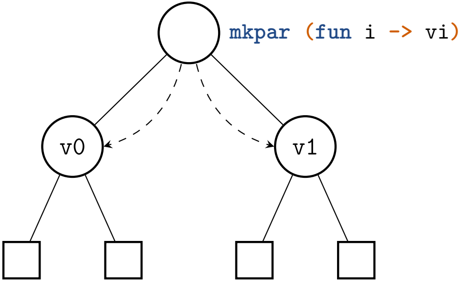
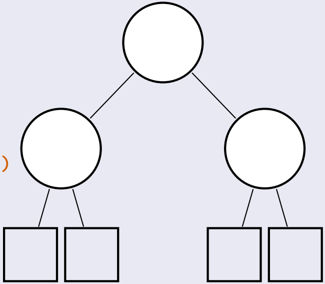


Code example

Keep the intermediate results of the sum

```
let multi tree sum_list l =
     where node =
        let v = mkpar (fun i -> split i l) in
        let rc = << sum_list $v$ >> in
        let s = sumSeq (flatten << at $rc$ >>)
        in finally rc s
     where leaf =

        sumSeq l
```



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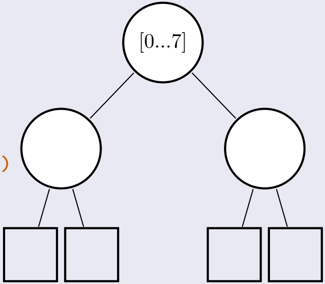

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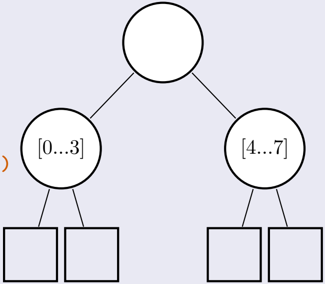

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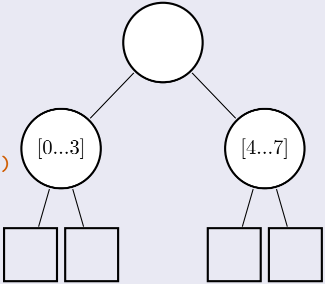

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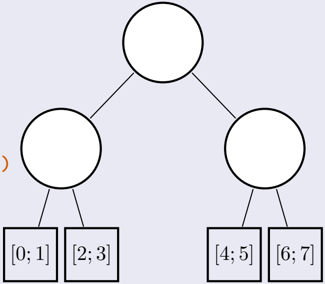

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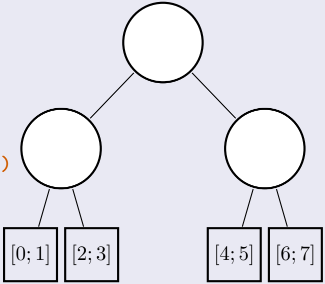

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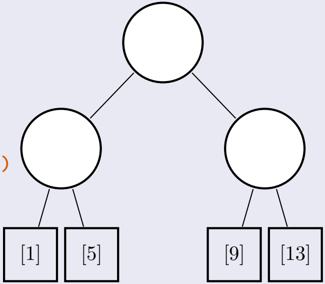

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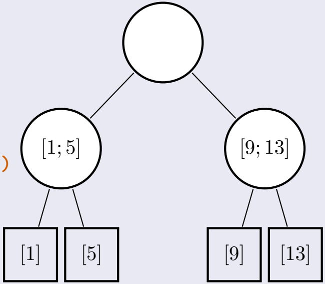

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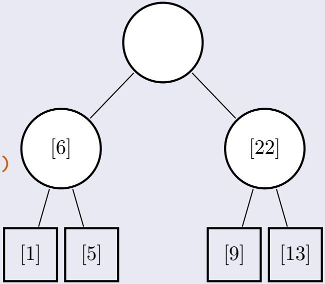

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Keep the intermediate results of the sum

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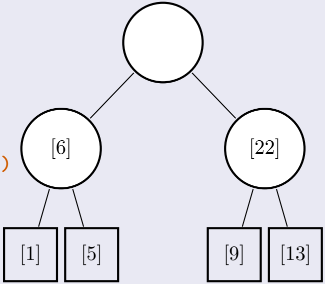
let multi tree sum_list l =
     where node =
        let v = mkpar (fun i -> split i l) in
        let rc = << sum_list $v$ >> in
        let s = sumSeq (flatten << at $rc$ >>)
  ──> in finally rc s
     where leaf =

        sumSeq l
```



Code example

Keep the intermediate results of the sum

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```

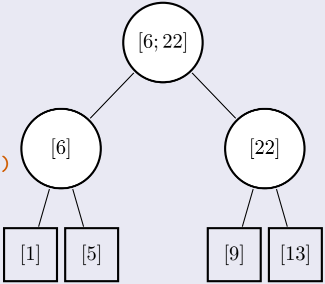

Code example

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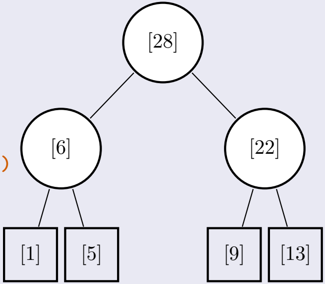

Code example

Keep the intermediate results of the sum

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let multi tree sum_list l =
     where node =
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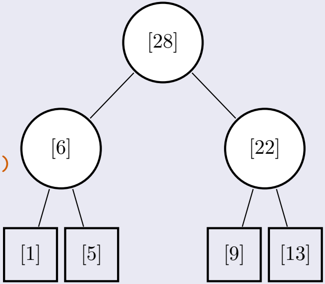

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2. The MULTI-ML language

3. Type system
   - Parallel program safety
   - The MULTI-ML typing system

4. Implementation

5. Conclusion

Parallel program safety

Parallel program safety

- Replicated coherency

Replicated coherency

```
if random_bool () then
  multi_fct ()
else
  (fun _ -> ...) ()
```

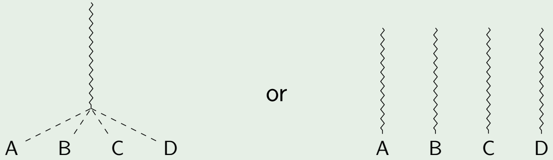
Parallel program safety

Parallel program safety

- Replicated coherency

Replicat

Why ?



Type system

Parallel program safety

- Replicated coherency
- Level (memory) compatibility

Level(memory) compatibility

```
<< let multi f x = ... >>
let x = #y#
let z = $v$
```

Type system

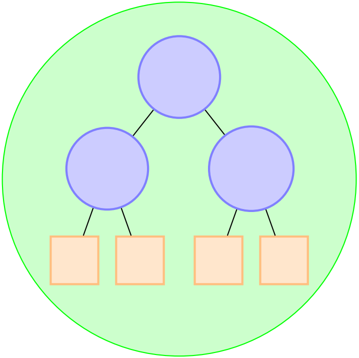
Parallel program safety

- Replicated coherency
- Level (memory) compatibility
- Control parallel structure imbrication
  - vector
  - tree

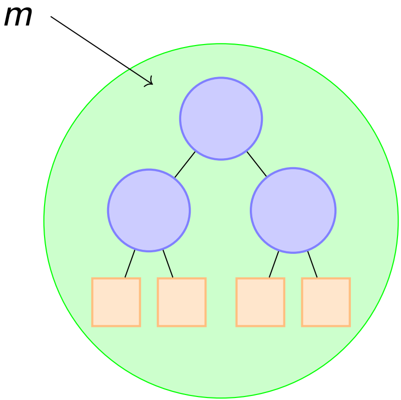
Parallel structure imbrication

```
<< let v = << 1 >> in v >>
let v = << 1 >> in << v >>
```

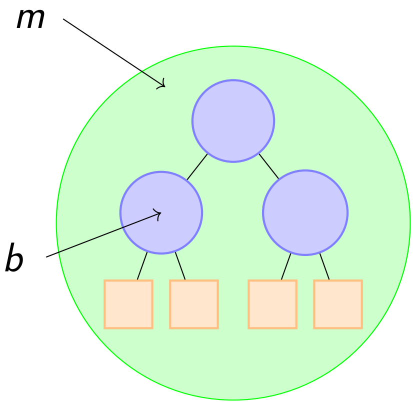
Type localities



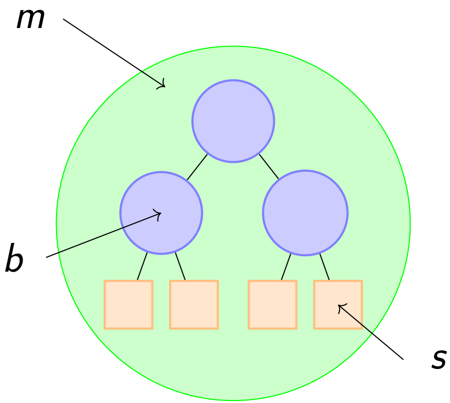
Type localities



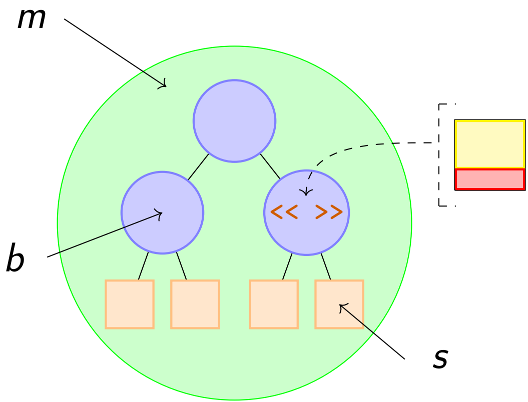
Type localities



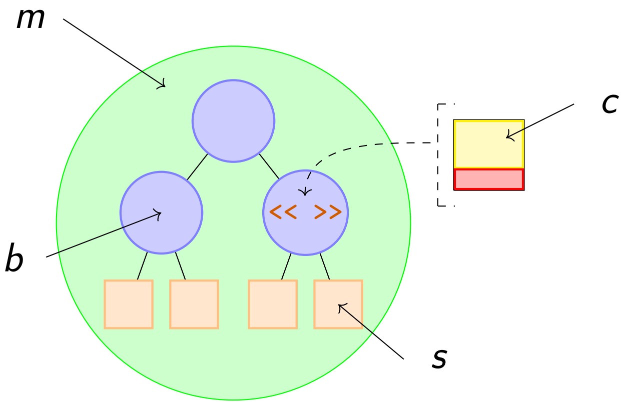
Type localities



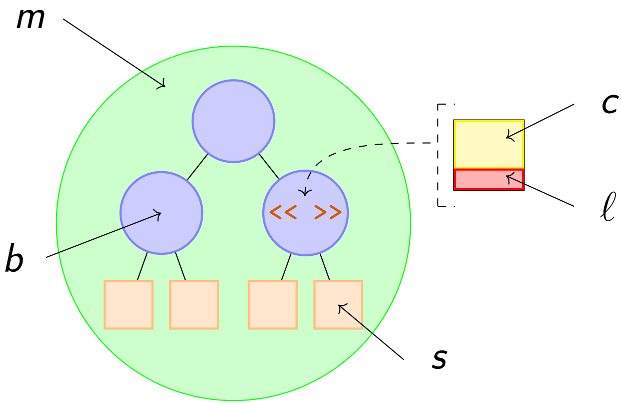
Type localities



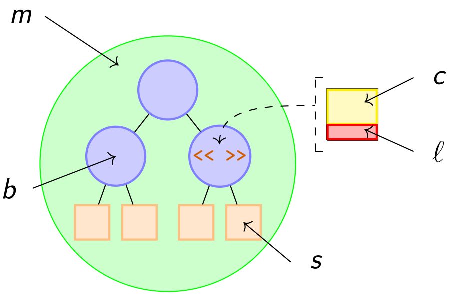
Type localities



Type localities



Type annotations



Type grammar

$$
\begin{array}{lll}
\tau & ::= & \\
 & \alpha_\pi & \textit{type variable} \\
 & \texttt{Base}_\pi & \textit{base type} \\
 & (\tau, \tau)_\pi & \textit{pair} \\
 & \tau\ \texttt{Par}_b & \textit{vector} \\
 & \tau\ \texttt{Tree}_\pi & \textit{tree} \\
 & (\tau \xrightarrow{\pi} \tau)_\pi & \textit{arrow type}
\end{array}
$$

$$\pi \quad ::= m \mid b \mid c \mid l \mid s$$

Type annotations

Latent effect

$$(\tau \xrightarrow{\pi} \tau)_{\pi'}$$

Where π is the effect *emitted* by the evaluation
and π' the locality of definition.

A BSP function

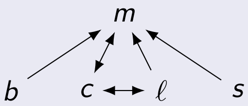
```
#let f = fun x ->
  let v = << ... >> in 1
-: val f : ('a_`z -(b)-> int_b)_m
```

$$f : ({}'a_{`z} \xrightarrow{b} int_b)_m$$

Accessibility

Accessibility: $\lhd$

$$\begin{array}{rcl} m, c & \lhd & m \\ m, b & \lhd & b \\ m, l, c & \lhd & l \\ m, l, c & \lhd & c \\ m, s & \lhd & s \end{array}$$

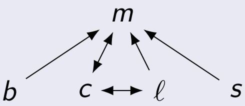


$\lambda_2 \lhd \lambda_1$: « λ_1 *can* read *in* λ_2 *memory.* »

Accessibility

Accessibility: $\lhd$

$$\begin{array}{rcl} m, c & \lhd & m \\ m, b & \lhd & b \\ m, l, c & \lhd & l \\ m, l, c & \lhd & c \\ m, s & \lhd & s \end{array}$$



$$\lambda_2 \lhd \lambda_1 : \text{« } \lambda_1 \textit{ can } \text{read} \textit{ in } \lambda_2 \textit{ memory. »}$$

Example:

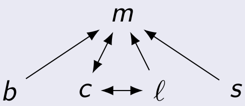
$$f : ({}'a_z \xrightarrow{b} int_b)_m$$

$$\texttt{f 1} \rightsquigarrow b \lhd m$$

Accessibility

Accessibility: ◁

$$\begin{aligned} m, c &\lhd m \\ m, b &\lhd b \\ m, l, c &\lhd l \\ m, l, c &\lhd c \\ m, s &\lhd s \end{aligned}$$



$\lambda_2 \lhd \lambda_1$: « λ_1 *can* read *in* λ_2 *memory.* »

Example:

$$f : ({'a}_z \xrightarrow{b} int_b)_m$$

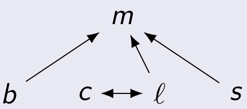
$$\texttt{f}\ 1 \quad \rightsquigarrow b \lhd m$$

Error

Definability

Definability: $\blacktriangleleft$

$$\begin{array}{rcl}
s, b, m & \blacktriangleleft & m \\
b & \blacktriangleleft & b \\
l, c & \blacktriangleleft & c \\
l, c & \blacktriangleleft & l \\
s & \blacktriangleleft & s
\end{array}$$

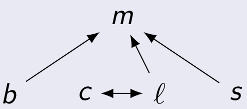


$\lambda_1 \blacktriangleleft \lambda_2$: « λ_1 *can be* defined *in* λ_2 *memory.* »

Definability

Definability: ◀

$$\begin{array}{rcl} s, b, m & \blacktriangleleft & m \\ b & \blacktriangleleft & b \\ l, c & \blacktriangleleft & c \\ l, c & \blacktriangleleft & l \\ s & \blacktriangleleft & s \end{array}$$



$\lambda_1 \blacktriangleleft \lambda_2$: « λ_1 *can be* defined *in* λ_2 *memory.* »

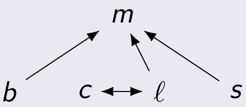
Example:

`<< let multi f x = ... >>` $\rightsquigarrow m \blacktriangleleft c$

Definability

Definability: $\blacktriangleleft$

$$\begin{array}{rcl}
s, b, m & \blacktriangleleft & m \\
b & \blacktriangleleft & b \\
l, c & \blacktriangleleft & c \\
l, c & \blacktriangleleft & l \\
s & \blacktriangleleft & s
\end{array}$$



$\lambda_1 \blacktriangleleft \lambda_2$: « λ_1 *can be* defined *in* λ_2 *memory.* »

Example:

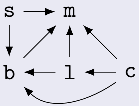
`<< let multi f x = ... >>` $\rightsquigarrow m \blacktriangleleft c$

Error

Other relations

Propagation

This relation returns the prevailing effect among ε and ε'.



Serialisation

Is it safe to communicate τ_π to locality Λ ?

$$\begin{array}{lcl}
\mathbf{Seria}_\Lambda(\mathtt{Base}_\pi) & = & \mathtt{Base}_\Lambda \ \textit{ if } \mathtt{Base} = \mathtt{int}, \mathtt{string}, \mathtt{float}, \mathtt{Bool}, \ldots \\
\mathbf{Seria}_\Lambda(\mathtt{Base}_\pi) & = & \mathtt{Fail} \ \textit{ if } \mathtt{Base} = \mathtt{i/o}, \ldots \\
\mathbf{Seria}_\Lambda(\tau_\pi) & = & \begin{cases} \tau_\Lambda, & \textit{if } \pi \lhd \Lambda \\ \mathtt{Fail}, & \textit{otherwise} \end{cases} \\
\mathbf{Seria}_\Lambda(\tau_\pi \ \mathbf{par}_\mathtt{b}) & = & \mathtt{Fail}
\end{array}$$

Formal properties

Operational semantics

Formal properties

Operational semantics

- Big Step semantics (deterministic)

Formal properties

Operational semantics

- Big Step semantics (deterministic)
- Big Step semantics for diverging terms (mutually exclusive)

Formal properties

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- Big Step semantics (deterministic)
- Big Step semantics for diverging terms (mutually exclusive)
- Programs that *"do not go wrong"* : $(\exists v. \Downarrow_p^{\mathcal{L}} v)$ or $(\Downarrow_p^{\mathcal{L}} \infty)$

Formal properties

Operational semantics

- Big Step semantics (deterministic)
- Big Step semantics for diverging terms (mutually exclusive)
- Programs that *"do not go wrong"* : $(\exists v. \Downarrow_{p}^{\mathcal{L}} v)$ or $(\Downarrow_{p}^{\mathcal{L}} \infty)$

Type safety of a MULTI-ML program

- Let e be an expression,
- Γ a typing environment,
- and c a set of constraint.

Then: $\Gamma \vdash e : \tau_{\pi}/\varepsilon[c]$ implies that e *"does not go wrong"* $(e \Rightarrow_{safe})$

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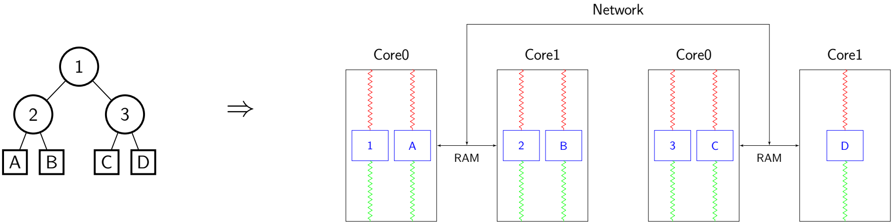
   - Execution scheme
   - Parallel and sequential implementations
   - Benchmarks

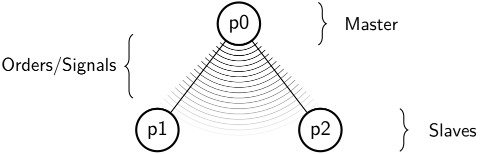
5. Conclusion

Execution scheme

- One process per leaf/node
- Distributed over physical cores



Execution scheme



p0

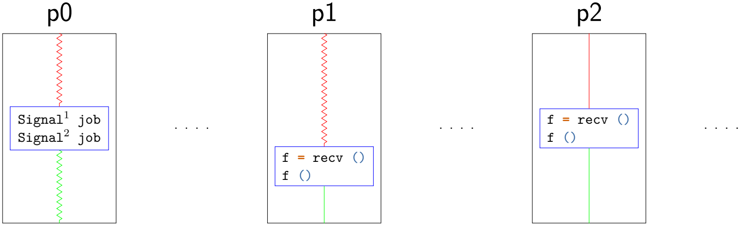
Master

Orders/Signals

p1

p2

Slaves



p0

```
Signal¹ job
Signal² job
```

. . . .

p1

```
f = recv ()
f ()
```

. . . .

p2

```
f = recv ()
f ()
```

. . . .

Formal properties

Correctness of a MULTI-ML program

If $e \Rightarrow_{safe}$ and $\mathcal{WF}(e)$ we have : $\langle\langle [\![e]\!]_{\mathrm{M}}, ..., [\![e]\!]_{\mathrm{M}} \rangle\rangle \Rightarrow_{safe}$

Distributed implementation

Module

- Communication library
- Based on operational semantics

Current implementation

- MPI processes
- Distributed over physical cores
- Shared/Distributed memory

Future implementations

- TCP/IP
- PTHREAD
- ...

Sequential implementation

Sequential simulator

- OCAML-like toplevel
- Test and debug
- Tree structure
- Hash tables to represent memories

```
#let multi tree f n =
  where node =
     let r =<<f ($pid$ + #n# + 1) >> in
       finally r (gid^"=>"^n)
  where leaf=
     (gid^"=>"^n);;

- : val f : int -> string tree = <multi-fun>
# (f 0)
o "0->0"
|
--o "0.0->1"
| |--> "0.0.0-> 2"
| |--> "0.0.1-> 3"
--o "0.1->2"
| |--> "0.1.0-> 3"
| |--> "0.1.1-> 4"
```

Benchmarks

Naive Eratosthenes sieve

- $\sqrt(n)$th first prime numbers
- Based on scan
- Unbalanced

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Naive Eratosthenes sieve

- $\sqrt(n)$th first prime numbers
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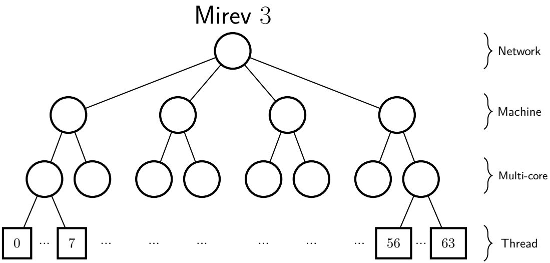
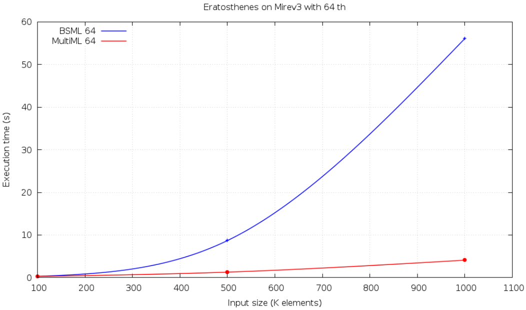


Benchmarks

Naive Eratosthenes sieve

- $\sqrt(n)$th first prime numbers
- Based on scan
- Unbalanced



Results

	100_000		500_000		1_000_000	
	MULTI-ML	BSML	MULTI-ML	BSML	MULTI-ML	BSML
8	0.7	1.8	22.4	105.0	125.3	430.7
64	0.3	0.3	1.3	8.7	4.1	56.1

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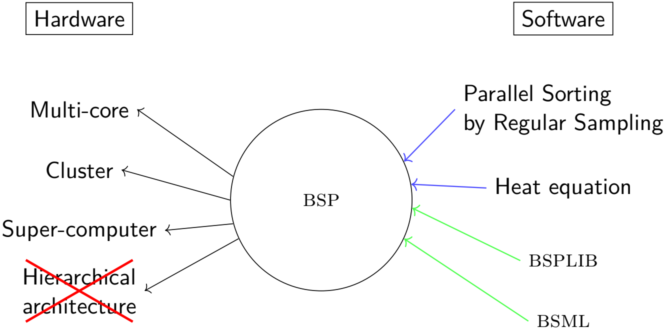
Before ...

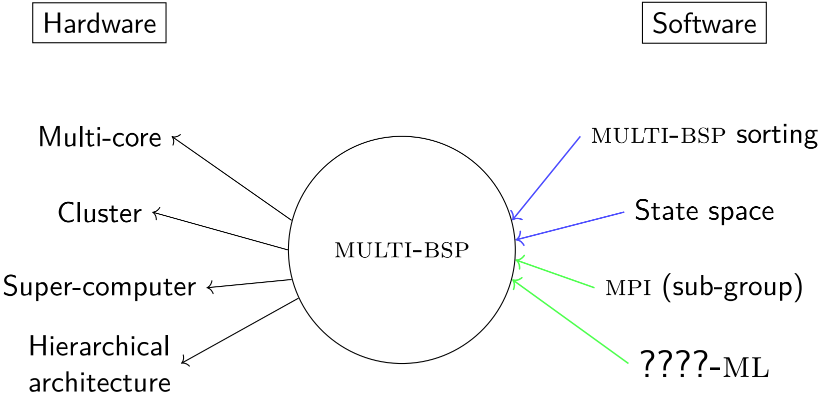
- BSP ≠ Hierarchical architecture
- BSML → BSP à la ML
- No language dedicated to MULTI-BSP



… Now

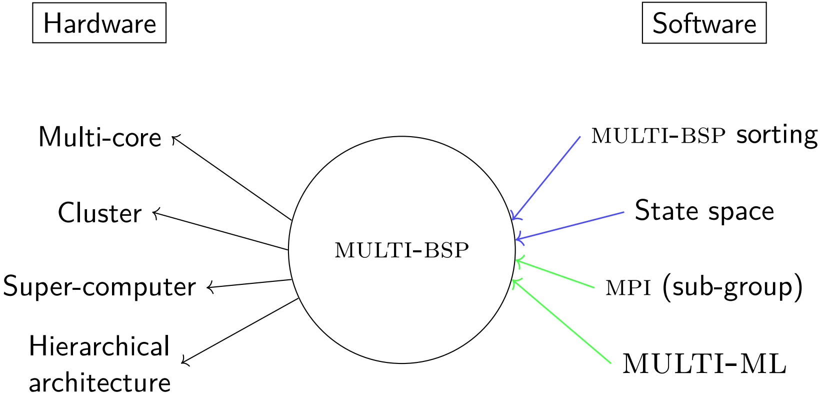
- MULTI-BSP extension of ML
  - Recursive multi-functions `let multi f x = ...`
  - BSML like code `where node = << f ... >>`
  - Small syntax extension `#,$,at,mkpar,finally,mktree,...`
- Type system
  - Constraints
  - Effects
- Operational semantics (even for diverging terms)
- Compilation scheme
- ⇒ Type safety from programs to abstract machines

Before ...



Hardware
Software
Multi-core
Cluster
Super-computer
Hierarchical architecture
MULTI-BSP
MULTI-BSP sorting
State space
MPI (sub-group)
????-ML

… Now



Hardware

Software

Multi-core

Cluster

Super-computer

Hierarchical architecture

MULTI-BSP

MULTI-BSP sorting

State space

MPI (sub-group)

MULTI-ML

Future Work

Ongoing work

- Code examples
- Extensions
  - Language
  - Type system

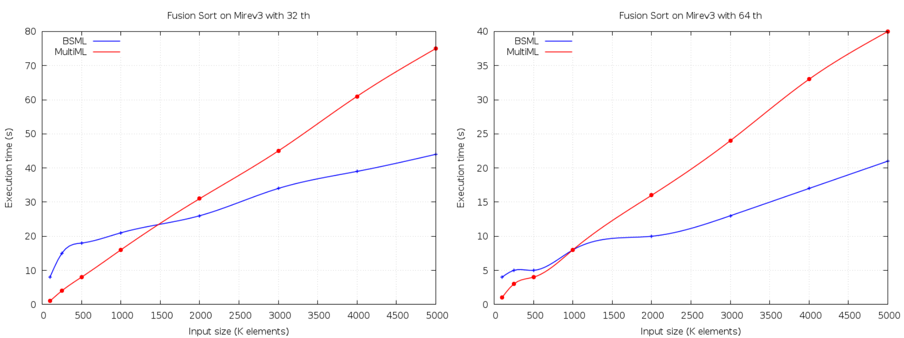
Future work

- MULTI-ML + GPU ⇒ Hybrid architectures
- Cost analysis
- Certified parallel programming

Thank you for your attention ☺

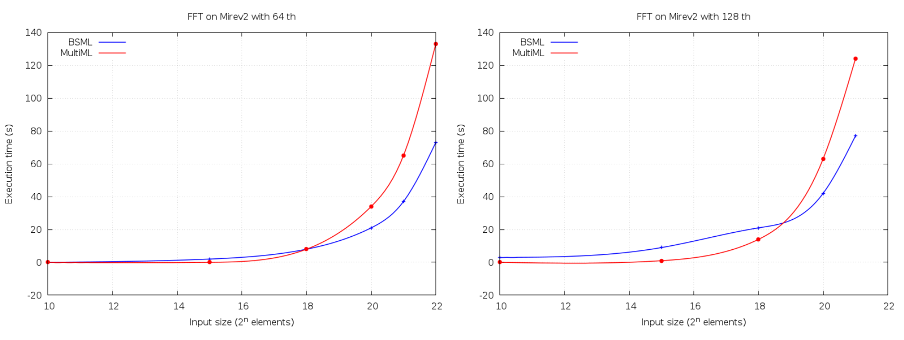
Questions ?

Fusion Sort on Mirev3 with 32 and 64 threads



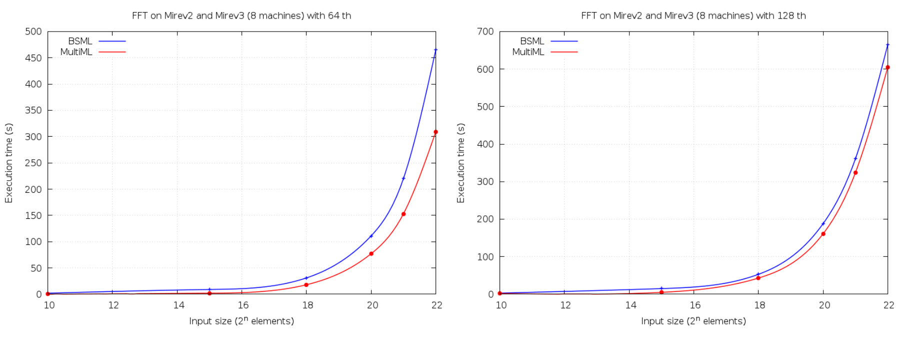
Fast Fourier Transform

FFT on Mirev2 (8 machines) with 64 and 128 threads



Fast Fourier Transform

FFT on Mirev2 (6 machines) and Mirev3 (2 machines) with 64 and 128 threads



Typing rules

$$\textsc{Let\ \ In}\qquad \frac{\begin{array}{l} \Lambda, \Gamma \vdash e_1 : \tau^1_{\pi_1} / \varepsilon_1 \ \ [c_1] \\ \Lambda, \Gamma ; x : \mathbf{Weak}(\tau^1_{\pi_1}, \varepsilon_1) \vdash e_2 : \tau^2_{\pi_2} / \varepsilon_2 \ \ [c_2] \\ c_3 \equiv [\Psi = \mathbf{Propgt}(\varepsilon_1, \varepsilon_2), c_1, c_2] \end{array}}{\Lambda, \Gamma \vdash \mathbf{let} \ \ x = e_1 \ \ \mathbf{in} \ \ e_2 : \tau^2_{\pi_2} / \Psi \ \ [c_3]}$$

```
<< fun _ -> let x = at t in x >>
```

```
<< let x = at t in fun _ -> x >>
```