

PROGRAMMING IN MULTI-BSP: MULTI-ML AND ITS TYPING SYSTEM

VICTOR ALLOMBERT, FRÉDÉRIC GAVA AND JULIEN TESSON

Laboratory of Algorithmic Complexity and Logic
Université Paris-Est

GDR LTP - LRI



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- ② Multi-ML in a nutshell
- ③ Multi-ML type system
- ④ Conclusion

Table of Contents

① Introduction

BSP

BSML

MULTI-BSP

② Multi-ML in a nutshell

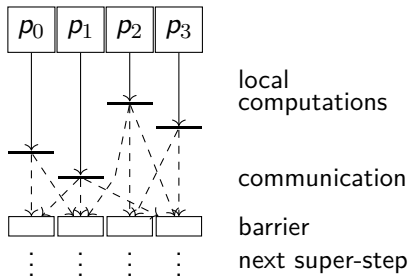
③ Multi-ML type system

④ Conclusion

Bulk Synchronous Parallelism

A BSP computer:

- p pairs CPU/memory
- Communication network
- Synchronization unit



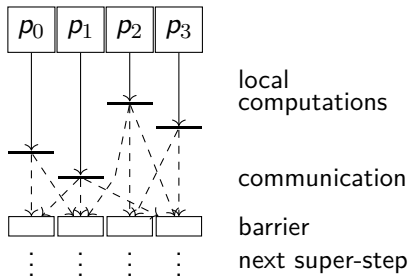
Bulk Synchronous Parallelism

A BSP computer:

- p pairs CPU/memory
- Communication network
- Synchronization unit

Properties:

- Super-steps execution
- Confluent
- Deadlock-free
- Predictable performances



Bulk Synchronous ML

What is BSML?



Bulk Synchronous ML

What is BSML?

- Explicit BSP programming with a functional approach



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- Based upon ML an implemented over OCAML
- Formal semantics \rightarrow computer-assisted proofs (COQ)



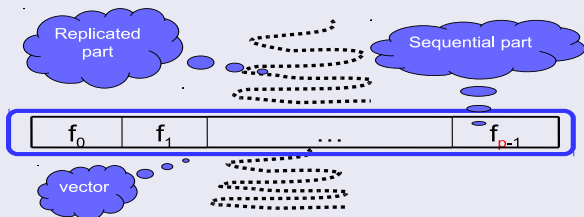
Bulk Synchronous ML

What is BSML?

- Explicit BSP programming with a functional approach
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Main idea

Parallel data structure \Rightarrow Vector:



BSML primitives

Asynchronous primitives

Asynchronous primitives

- `<< e >>` : $\langle e, \dots, e \rangle$

Asynchronous primitives

- $\langle\langle e \rangle\rangle$: $\langle e, \dots, e \rangle$
- $\$v\$$: v_i on processor i , assumes $v \equiv \langle v_0, \dots, v_{p-1} \rangle$

Asynchronous primitives

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Synchronous primitives

- `proj` : $\langle x_0, \dots, x_{p-1} \rangle \mapsto (\text{fun } i \rightarrow x_i)$
- `put` : $\langle f_0, \dots, f_{p-1} \rangle \mapsto \langle (\text{fun } i \rightarrow f_i 0), \dots, (\text{fun } i \rightarrow f_i (p-1)) \rangle$

Code example

For a BSP machine with 3 processors:

```
# let vec = << "Hello" >>;;
val vec : string par = <"Hello", "Hello", "Hello">

# let vec2 = << $vec$^(string_of_int $pid$) >>;;
val vec2 : string par = <"Hello0", "Hello1", "Hello2">

# let totex v = List.map (proj v) procs;;
val totex : 'a par -> 'a list = <fun>

# totex vec2;;
- : string list = ["Hello0"; "Hello1"; "Hello2"]
```

The MULTI-BSP model

What is MULTI-BSP?

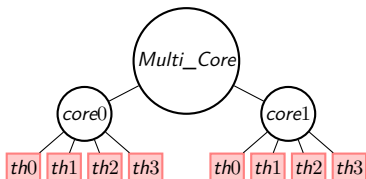
- ① A tree structure with nested components
- ② Where nodes have a storage capacity
- ③ And leaves are processors

The MULTI-BSP model

What is MULTI-BSP?

- 1 A tree structure with nested components
- 2 Where nodes have a storage capacity
- 3 And leaves are processors

MULTI-BSP

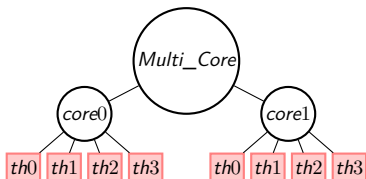


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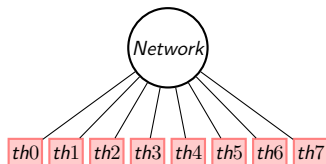
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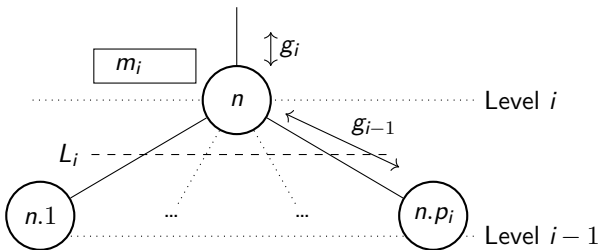
BSP



The MULTI-BSP model

Execution model

A level i superstep is:

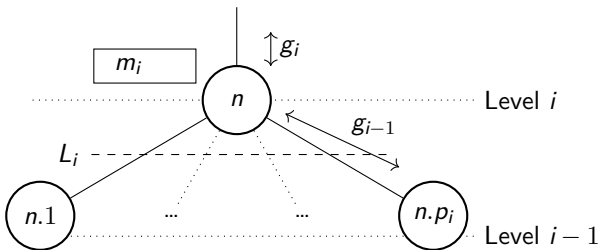


The MULTI-BSP model

Execution model

A level i superstep is:

- Level $i-1$ executes code independently

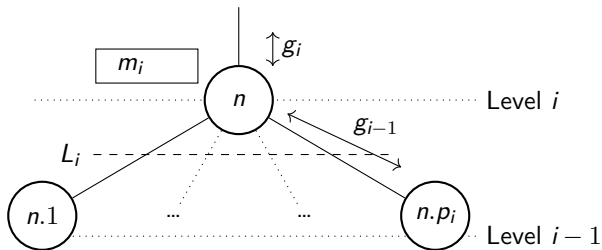


The MULTI-BSP model

Execution model

A level i superstep is:

- Level $i - 1$ executes code independently
- Exchanges informations with the m_i memory



The MULTI-BSP model

Execution model

A level i superstep is:

- Level $i - 1$ executes code independently
- Exchanges informations with the m_i memory
- Synchronises

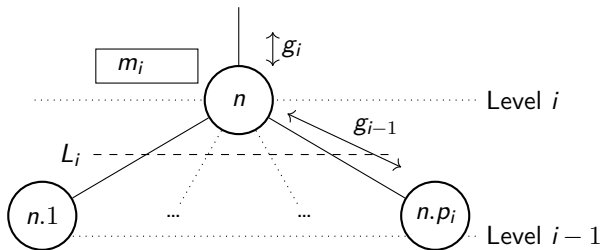


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Basic ideas

Basic ideas

- BSML-like code on every stage of the MULTI-BSP architecture

Basic ideas

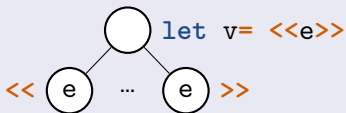
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- *Multi-functions* that recursively go through the tree.

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- Specific syntax over ML: eases programming
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MULTI-ML: Tree recursion

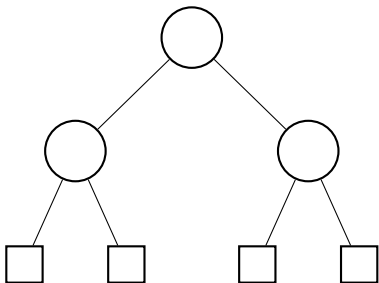
Recursion structure

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let multi f [args]=  
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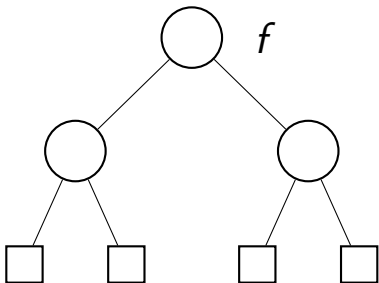
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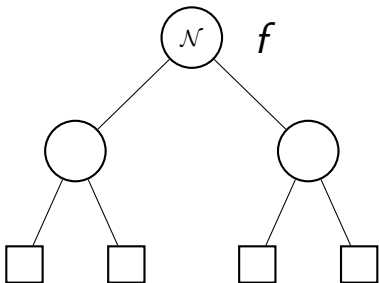
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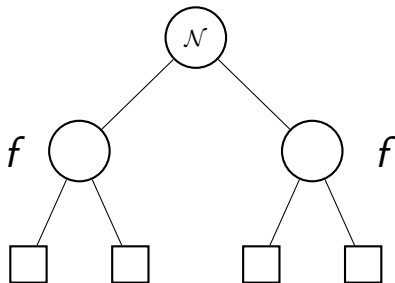
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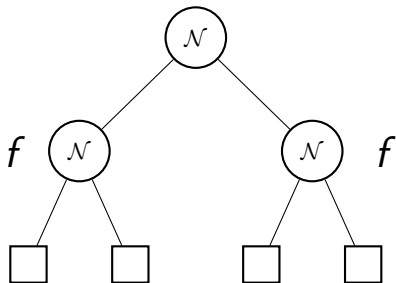
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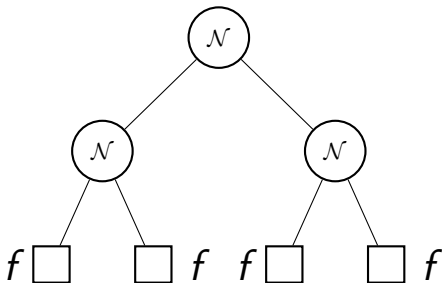
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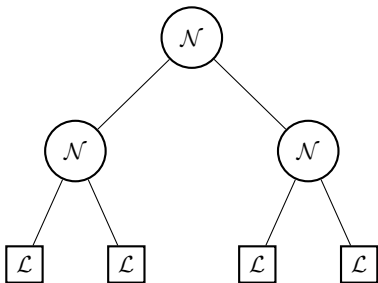
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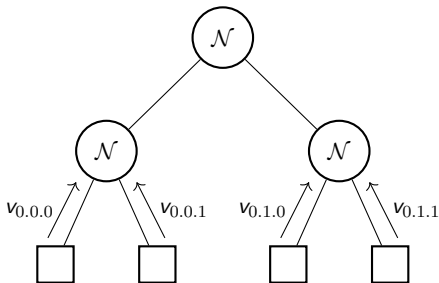
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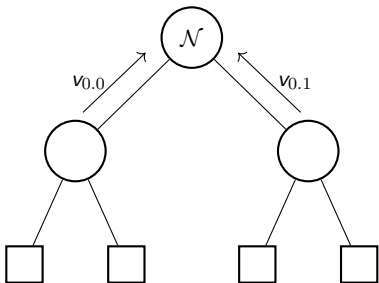
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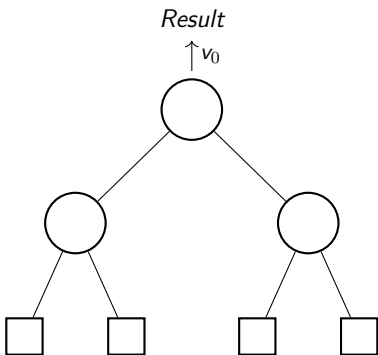
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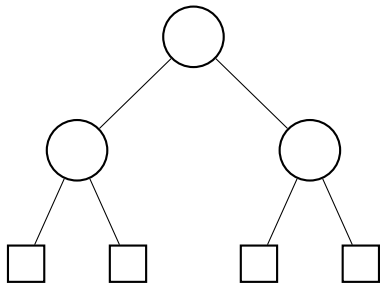
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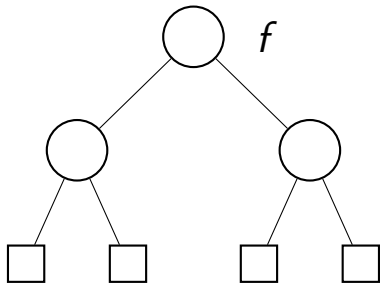
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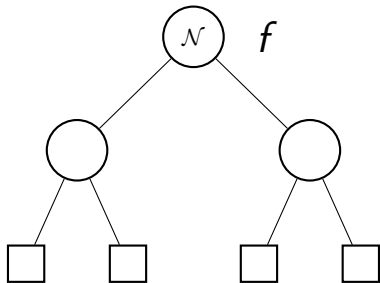
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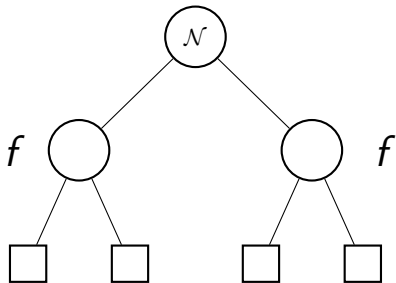
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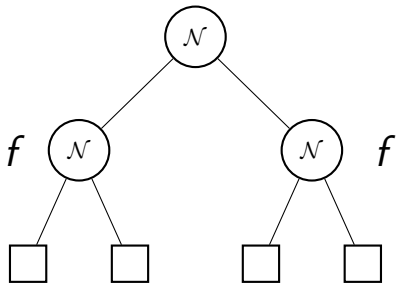
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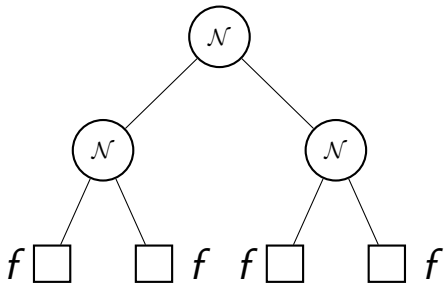
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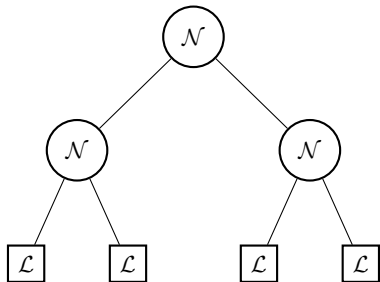
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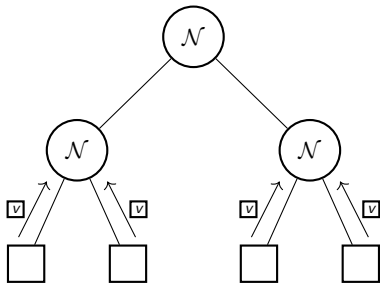
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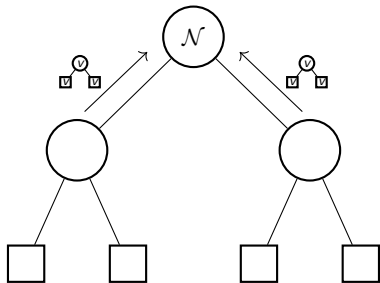
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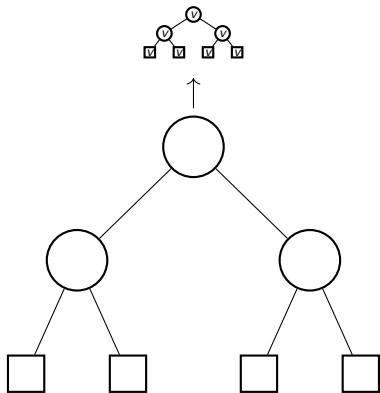
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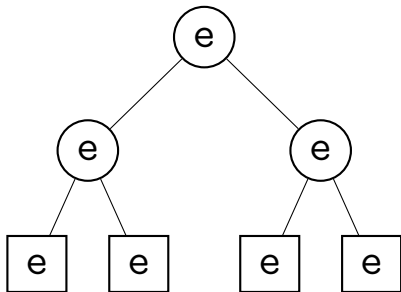
Primitives

Summary

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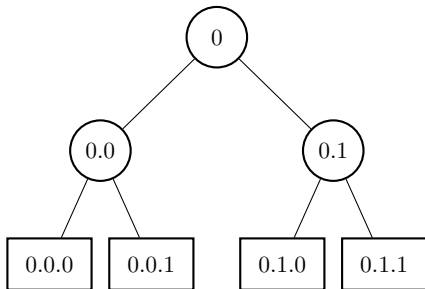
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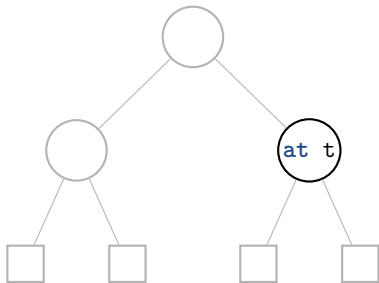
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Primitives

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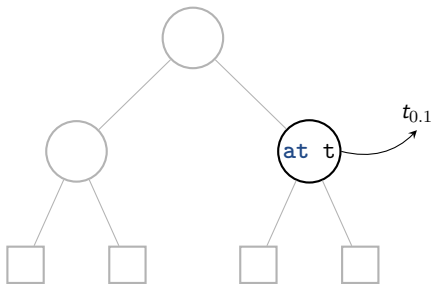
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Primitives

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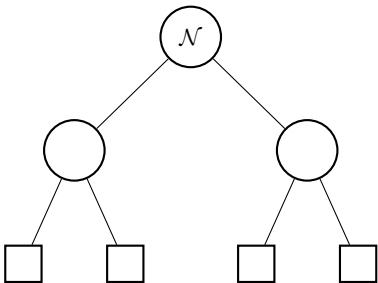
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Primitives

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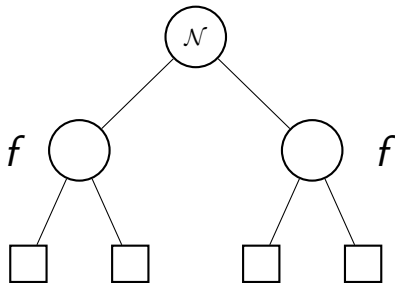
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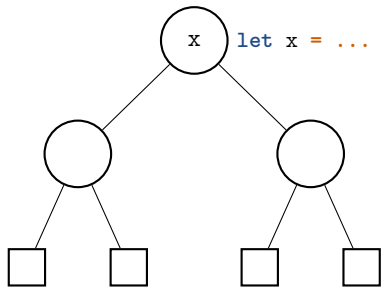
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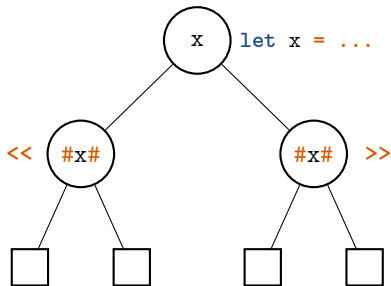
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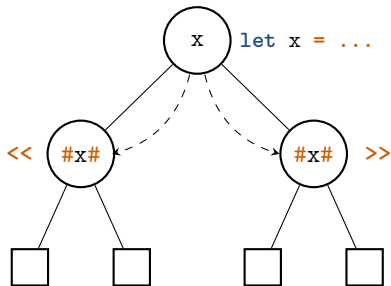
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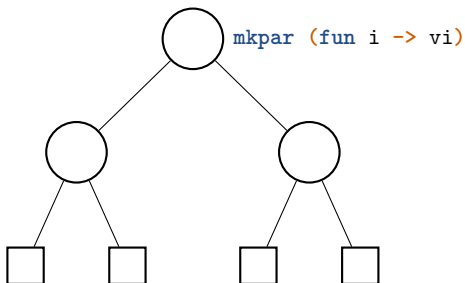
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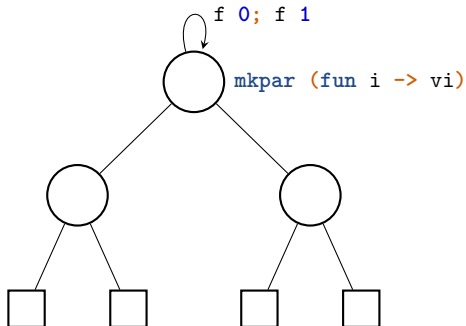
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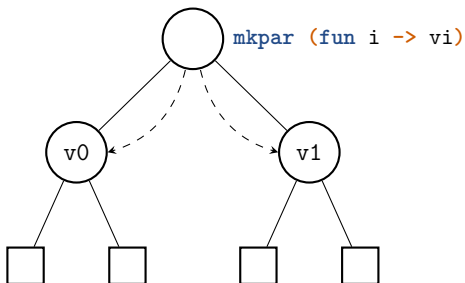
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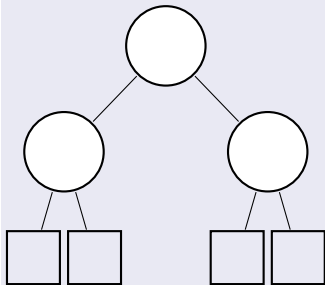
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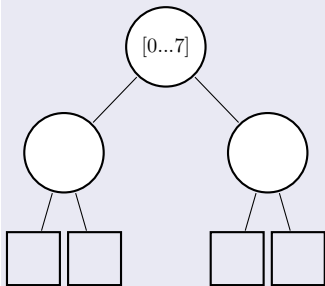
Keep the intermediate results of the sum



```
let multi tree sum_list l =  
  where node =  
    let v = mkpar (fun i -> split i l) in  
    let rc = << sum_list $v$ >> in  
    let s = sumSeq (flatten << at $rc$ >>)  
    in (rc,s)  
  where leaf =  
    sumSeq l
```

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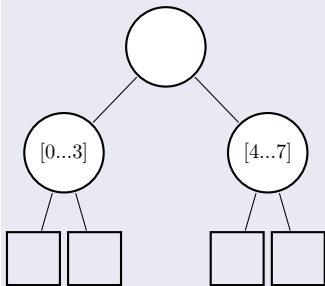
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    let s = sumSeq (flatten << at $rc$ >>)  
    in (rc,s)  
  where leaf =  
    sumSeq l
```

Code example

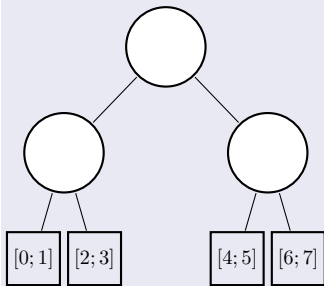
Keep the intermediate results of the sum



```
let multi tree sum_list l =  
  where node =  
    let v = mkpar (fun i -> split i l) in  
    let rc = << sum_list $v$ >> in  
    let s = sumSeq (flatten << at $rc$ >>)  
    in (rc,s)  
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```

Code example

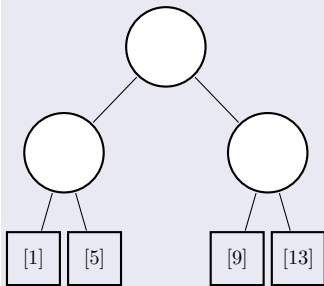
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```

Code example

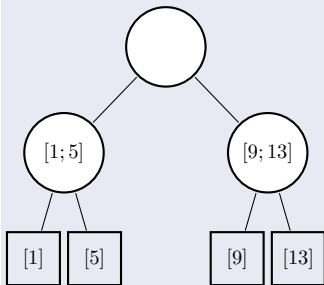
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    in (rc,s)
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Code example

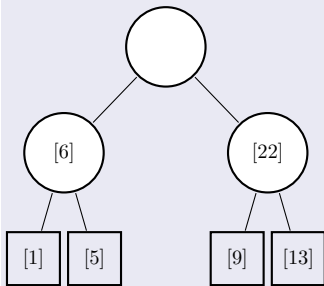
Keep the intermediate results of the sum



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Code example

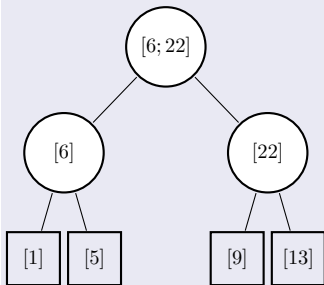
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```


Code example

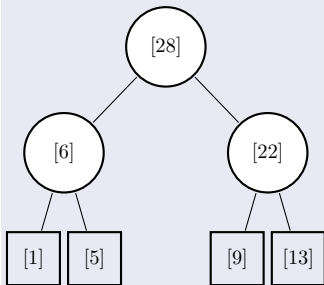
Keep the intermediate results of the sum



```
let multi tree sum_list l =  
  where node =  
    let v = mkpar (fun i -> split i l) in  
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Code example

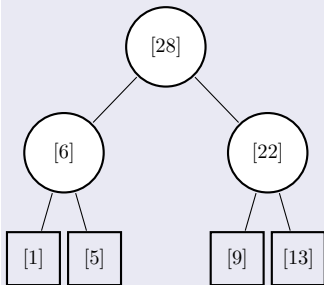
Keep the intermediate results of the sum



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  where leaf =
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Code example

Keep the intermediate results of the sum



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let multi tree sum_list l =
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    let rc = << sum_list $v$ >> in
    let s = sumSeq (flatten << at $rc$ >>)
    in (rc,s)
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```

Implementation

Run on multi-core clusters using MPI.

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- ① Introduction
- ② Multi-ML in a nutshell
- ③ Multi-ML type system**
- ④ Conclusion

Parallel program safety

- Replicated coherency

Replicated coherency

```
if random_bool () then
  proj v
else
  ...
```

Parallel program safety

- Replicated coherency
- Level (memory) compatibility

Level(memory) compatibility

```
<< let multi f x = ... >>
```

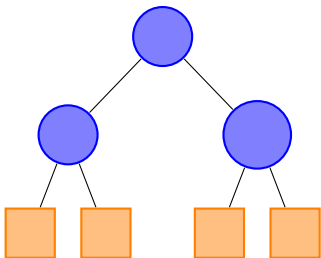
Parallel program safety

- Replicated coherency
- Level (memory) compatibility
- Control parallel structure imbrication
 - vector
 - tree

Parallel structure imbrication

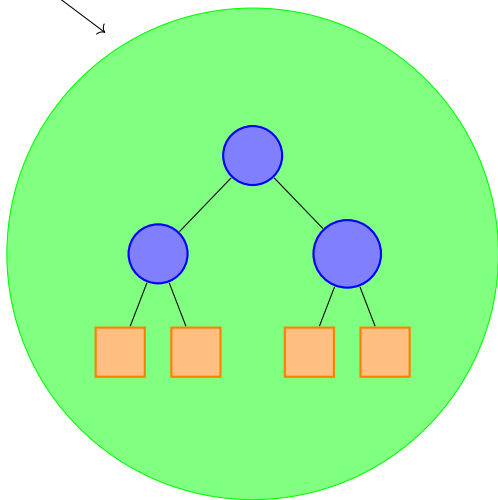
```
let v = << ... >> in << v >>
```

Type localities

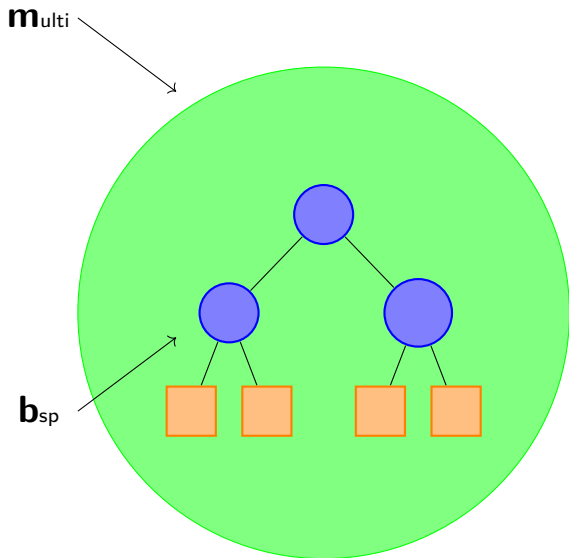


Type localities

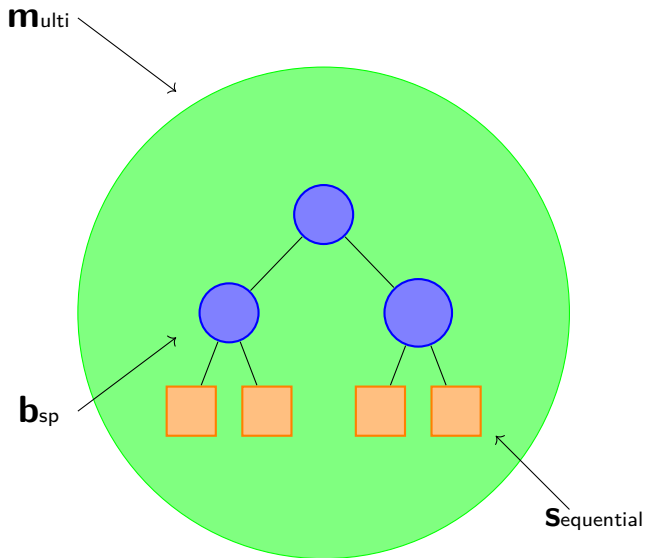
m_{multi}



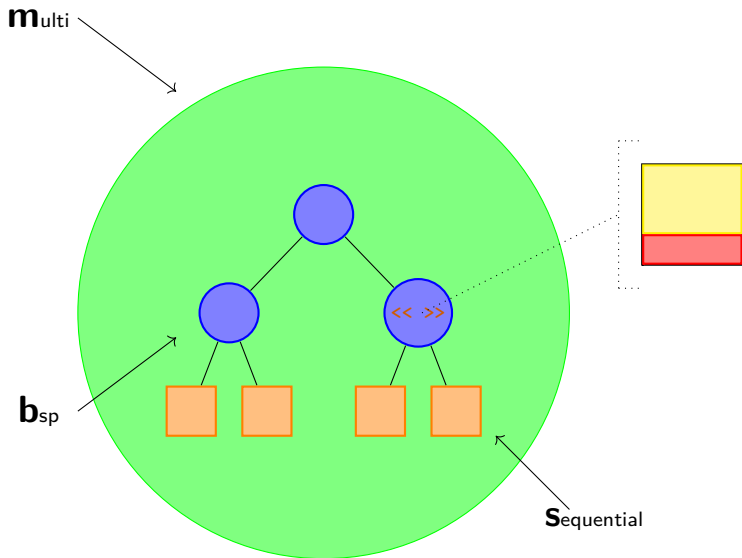
Type localities



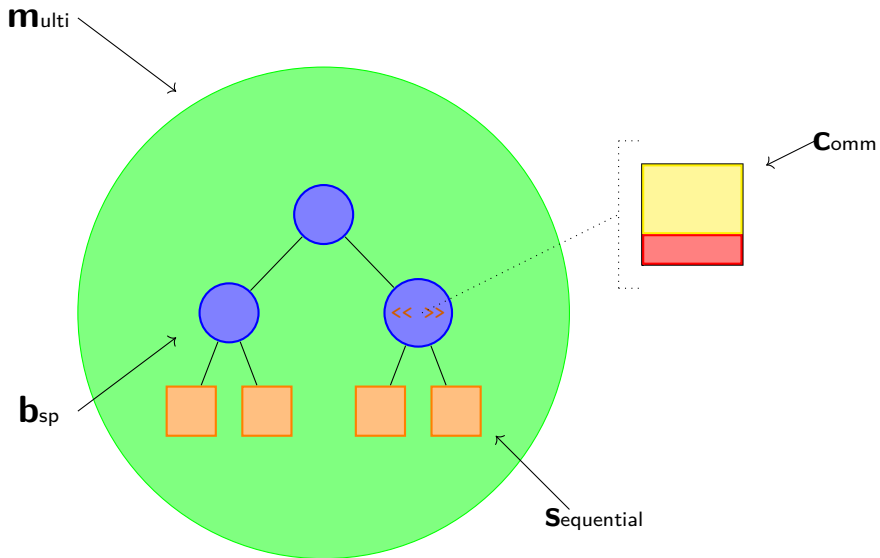
Type localities



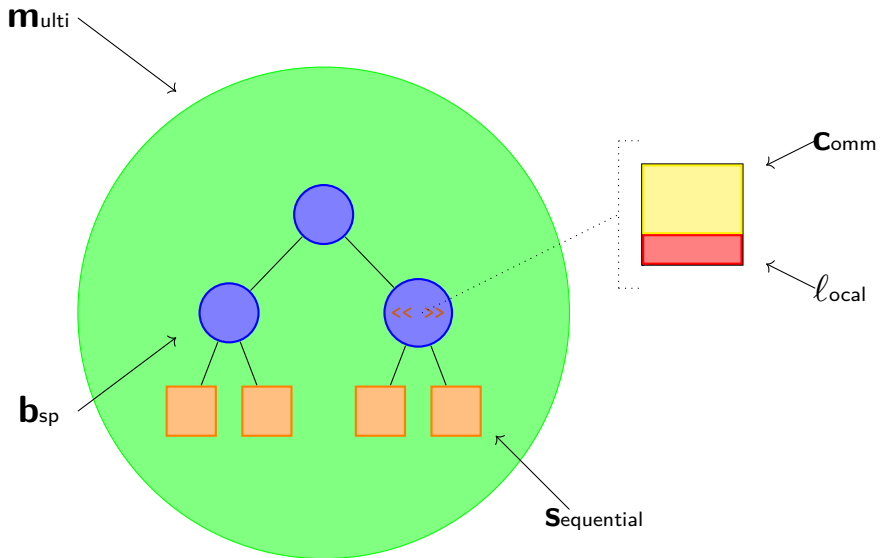
Type localities



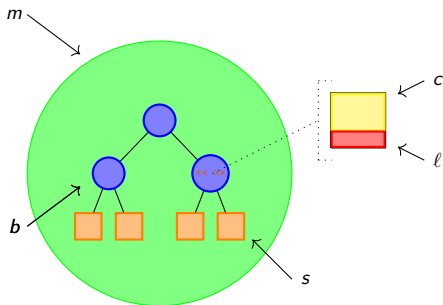
Type localities



Type localities



Type annotations



Type annotation

τ	$::=$	
α_π		<i>type variable</i>
Base_π		<i>base type</i>
$(\tau, \tau)_\pi$		<i>pairs</i>
$\tau \text{ Par}_b$		<i>vector</i>
$\tau \text{ Tree}_\pi$		<i>tree</i>
$(\tau \xrightarrow{\pi} \tau)_\pi$		<i>arrow type</i>

$\pi ::= m \mid b \mid c \mid l \mid s$

Latent effect

$$(\mathcal{T} \xrightarrow{\pi} \mathcal{T})_{\pi'}$$

Where π is the effect emitted by the evaluation and π' the locality of definition.

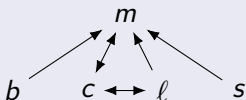
A BSP function

```
#let f = fun x ->  
  let v = << ... >> in 1  
-: val f : ('a_`z -(b)-> int_b)_m
```

$$f: ('a_`z \xrightarrow{b} int_b)_m$$

Accessibility: \triangleleft

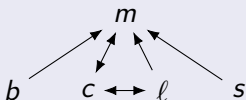
$m, c \triangleleft m$
 $m, b \triangleleft b$
 $m, l, c \triangleleft l$
 $m, l, c \triangleleft c$
 $m, s \triangleleft s$



$\lambda_2 \triangleleft \lambda_1$: « λ_1 can read in λ_2 memory. »

Accessibility: \triangleleft

$m, c \triangleleft m$
 $m, b \triangleleft b$
 $m, l, c \triangleleft l$
 $m, l, c \triangleleft c$
 $m, s \triangleleft s$



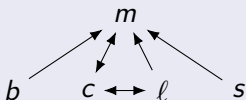
$\lambda_2 \triangleleft \lambda_1$: « λ_1 can read in λ_2 memory. »

Example:

$f: ('a_z \xrightarrow{b} \text{int}_b)_m$
 $f \ 1 \rightsquigarrow b \triangleleft m$

Accessibility: \triangleleft

$m, c \triangleleft m$
 $m, b \triangleleft b$
 $m, l, c \triangleleft l$
 $m, l, c \triangleleft c$
 $m, s \triangleleft s$



$\lambda_2 \triangleleft \lambda_1$: « λ_1 can read in λ_2 memory. »

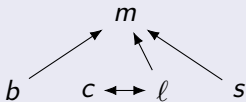
Example:

$f: ('a_z \xrightarrow{b} int_b)_m$
 $f \ 1 \rightsquigarrow b \triangleleft m$

Error

Definability: ◀

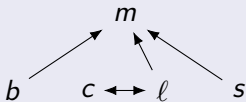
s, b, m ◀ m
 b ◀ b
 l, c ◀ c
 l, c ◀ l
 s ◀ s



λ_1 ◀ λ_2 : « λ_1 can be defined in λ_2 memory. »

Definability: ◀

$s, b, m \llcorner m$
 $b \llcorner b$
 $l, c \llcorner c$
 $l, c \llcorner l$
 $s \llcorner s$



$\lambda_1 \llcorner \lambda_2 : \ll \lambda_1 \text{ can be defined in } \lambda_2 \text{ memory. } \gg$

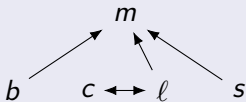
Example:

$\ll \text{let multi } f \ x = \dots \gg \rightsquigarrow m \llcorner b$

Definability

Definability: ◀

$s, b, m \llcorner m$
 $b \llcorner b$
 $l, c \llcorner c$
 $l, c \llcorner l$
 $s \llcorner s$



$\lambda_1 \llcorner \lambda_2 : \ll \lambda_1 \text{ can be defined in } \lambda_2 \text{ memory. } \gg$

Example:

$\ll \text{let multi } f \ x = \dots \gg \rightsquigarrow m \llcorner b$
Error

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

$Propgt$	m	b	l	c	s
m	m	m	m	m	m
b	m	b	b	b	b
l	m	b	l	l	\perp
c	m	b	l	c	\perp
s	m	b	\perp	\perp	s

Propagation

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

$Propgt$	m	b	l	c	s
m	m	m	m	m	m
b	m	b	b	b	b
l	m	b	l	l	\perp
c	m	b	l	c	\perp
s	m	b	\perp	\perp	s

Constraint generation

$$\llbracket e_1 \ e_2 : \tau_\Lambda / \Psi \rrbracket^\Lambda =$$

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

<i>Propgt</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>c</i>	<i>s</i>
<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>
<i>b</i>	<i>m</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>
<i>l</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>l</i>	\perp
<i>c</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>c</i>	\perp
<i>s</i>	<i>m</i>	<i>b</i>	\perp	\perp	<i>s</i>

Constraint generation

$$\llbracket e_1 e_2 : \tau_\Lambda / \Psi \rrbracket^\Lambda = \llbracket e_1 : (\alpha_\pi \xrightarrow{\varepsilon} \tau_{\pi'})_\delta / \varepsilon' \rrbracket^\Lambda$$

Propagation

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

<i>Propgt</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>c</i>	<i>s</i>
<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>
<i>b</i>	<i>m</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>
<i>l</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>l</i>	\perp
<i>c</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>c</i>	\perp
<i>s</i>	<i>m</i>	<i>b</i>	\perp	\perp	<i>s</i>

Constraint generation

$$\llbracket e_1 \ e_2 : \tau_\Lambda / \Psi \rrbracket^\Lambda = \llbracket e_1 : (\alpha_\pi \xrightarrow{\varepsilon} \tau_{\pi'})_\delta / \varepsilon' \rrbracket^\Lambda \wedge \llbracket e_2 : \alpha_{\pi''} / \varepsilon'' \rrbracket^\Lambda$$

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

<i>Propgt</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>c</i>	<i>s</i>
<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>	<i>m</i>
<i>b</i>	<i>m</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>
<i>l</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>l</i>	\perp
<i>c</i>	<i>m</i>	<i>b</i>	<i>l</i>	<i>c</i>	\perp
<i>s</i>	<i>m</i>	<i>b</i>	\perp	\perp	<i>s</i>

Constraint generation

$$\llbracket e_1 \ e_2 : \tau_\Lambda / \Psi \rrbracket^\Lambda = \llbracket e_1 : (\alpha_\pi \xrightarrow{\varepsilon} \tau_{\pi'})_\delta / \varepsilon' \rrbracket^\Lambda \wedge \llbracket e_2 : \alpha_{\pi''} / \varepsilon'' \rrbracket^\Lambda$$

$$\wedge \pi'' \triangleleft \pi$$

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

$Propgt$	m	b	l	c	s
m	m	m	m	m	m
b	m	b	b	b	b
l	m	b	l	l	\perp
c	m	b	l	c	\perp
s	m	b	\perp	\perp	s

Constraint generation

$$\llbracket e_1 \ e_2 : \tau_\Lambda / \Psi \rrbracket^\Lambda = \llbracket e_1 : (\alpha_\pi \xrightarrow{\varepsilon} \tau_{\pi'})_\delta / \varepsilon' \rrbracket^\Lambda \wedge \llbracket e_2 : \alpha_{\pi''} / \varepsilon'' \rrbracket^\Lambda$$

$$\wedge \pi'' \triangleleft \pi \ \wedge \varepsilon \triangleleft \Lambda$$

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

$Propgt$	m	b	l	c	s
m	m	m	m	m	m
b	m	b	b	b	b
l	m	b	l	l	\perp
c	m	b	l	c	\perp
s	m	b	\perp	\perp	s

Constraint generation

$$\llbracket e_1 \ e_2 : \tau_\Lambda / \Psi \rrbracket^\Lambda = \llbracket e_1 : (\alpha_\pi \xrightarrow{\varepsilon} \tau_{\pi'})_\delta / \varepsilon' \rrbracket^\Lambda \wedge \llbracket e_2 : \alpha_{\pi''} / \varepsilon'' \rrbracket^\Lambda$$

$$\wedge \pi'' \triangleleft \pi \quad \wedge \varepsilon \triangleleft \Lambda \quad \wedge \varepsilon \blacktriangleleft \Lambda$$

Propagation: $Propgt(\varepsilon, \varepsilon')$

This relation returns the prevailing effect amongst ε and ε' .

$Propgt$	m	b	l	c	s
m	m	m	m	m	m
b	m	b	b	b	b
l	m	b	l	l	\perp
c	m	b	l	c	\perp
s	m	b	\perp	\perp	s

Constraint generation

$$\llbracket e_1 \ e_2 : \tau_\Lambda / \Psi \rrbracket^\Lambda = \llbracket e_1 : (\alpha_\pi \xrightarrow{\varepsilon} \tau_{\pi'})_\delta / \varepsilon' \rrbracket^\Lambda \wedge \llbracket e_2 : \alpha_{\pi''} / \varepsilon'' \rrbracket^\Lambda$$

$$\wedge \pi'' \triangleleft \pi \wedge \varepsilon \triangleleft \Lambda \wedge \varepsilon \blacktriangleleft \Lambda \wedge \Psi = Propgt(\varepsilon, \varepsilon', \varepsilon'')$$

Serialisation: $Seria_{\Lambda}(\tau_{\pi})$

Is it safe to communicate τ_{π} to locality Λ ?

$$\begin{aligned}
 Seria_{\alpha}(\tau_{\pi}) &= \left\{ \tau_{\pi}, \quad \text{if } \pi \triangleleft \alpha \right. \\
 Seria_{\alpha}(\text{Base}_{\pi}) &= \text{Base}_{\pi} \quad \text{if Base} = \text{int, Bool, ...} \\
 Seria_{\alpha}(\text{Base}_{\pi}) &= \text{Fail} \quad \text{if Base} = \text{i/o, ...} \\
 Seria_{\alpha}(\tau_{\pi} \text{ par } b) &= \text{Fail} \\
 Seria_{\alpha}(\text{tree}_{\pi}) &= \text{Fail} \\
 Seria_{\alpha}(_ \xrightarrow{!} _) &= \text{Fail} \\
 \dots & \\
 Seria_{\alpha}((\tau_{\pi}^1 \xrightarrow{\varepsilon} \tau_{\pi'}^2)_{\delta}) &= \left\{ \begin{array}{l} (\tau_{\pi}^{\prime 1} \xrightarrow{\varepsilon} \tau_{\pi'}^{\prime 2})_{\delta}, \quad \text{if } \varepsilon \# m, b, s, l \\ \text{and } \tau_{\pi}^{\prime 1} = Seria_{\alpha}(\tau_{\pi}^1) \\ \text{and } \tau_{\pi'}^{\prime 2} = Seria_{\alpha}(\tau_{\pi'}^2) \end{array} \right.
 \end{aligned}$$

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MULTI-ML

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MULTI-ML

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- Big-steps formal semantics (confluent)
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Current/Future work

- Full implementation (record, tuples, ...)
- Variants
- Modules and other OCAML features
- Error tracking

Merci !

Any questions ?