Multi-ML: Programming Multi-BSP Algorithms in ML

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Journées du GDR GPL - Besançon

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Ocaml : a ML language

Strengths of Ocaml

- A functionnal programming language
- A powerful type system
- User-definable algebraic data types and pattern matching
- Automatic memory management
- Efficient native code compilers

Syntaxe overview

```
# let f = fun x -> "Hello "^(string_of_int x) in
  let lst = [0;1;2] in
  List.map f lst;;
- : string list = ["Hello 0"; "Hello 1"; "Hello 2"]

# let pair = ([0;1;2],true);;
val pair : int list * bool = ([0; 1; 2], true)

# type 'a list =
   Nil
   |  Node of 'a*'a list ;;
type 'a list = Nil | Node of 'a * 'a list
```

Bulk Synchronous ML

What is BSML?

Bulk Synchronous ML

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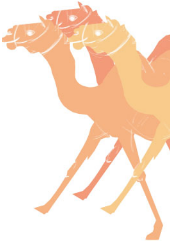
- Explicit BSP programming with a functional approach



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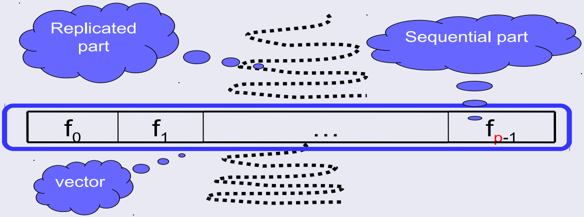
Main idea

Parallel data structure ⇒ Vector:



BSML primitives

Asynchronous primitives

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Synchronous primitives

- `proj` : $\langle x_0, \ldots, x_{\mathbf{p}-1} \rangle \mapsto (\textbf{fun } i \rightarrow x_i)$

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Synchronous primitives

- `proj` : $\langle x_0, \ldots, x_{\mathbf{p}-1} \rangle \mapsto (\textbf{fun } i \to x_i)$
- `put` : $\langle f_0, \ldots, f_{\mathbf{p}-1} \rangle \mapsto \langle (\textbf{fun } i \to f_i\ 0), \ldots, (\textbf{fun } i \to f_i\ (\mathbf{p}-1)) \rangle$

Code example

For a BSP machine with 3 processors:

```
# let vec = << "Hello" >>;;
val vec : string par = <"Hello", "Hello", "Hello">

# let vec2 = << $vec$^(string_of_int $pid$) >>;;
val vec2 : string par = <"Hello0", "Hello1", "Hello2">

# let totex v = List.map (proj v) procs;;
val totex : 'a par -> 'a list = <fun>

# totex vec2;;
- : string list = ["Hello0"; "Hello1"; "Hello2"]
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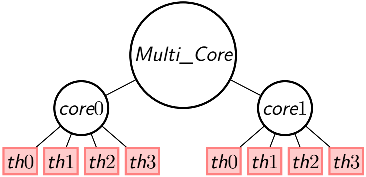

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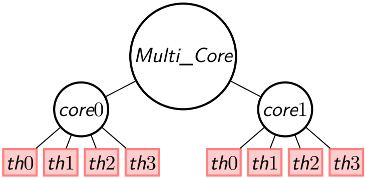
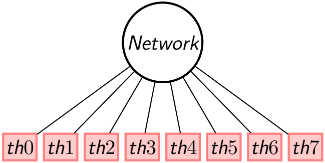


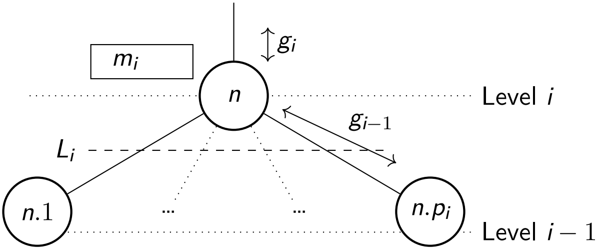
BSP



The MULTI-BSP model

Execution model

A level i superstep is:



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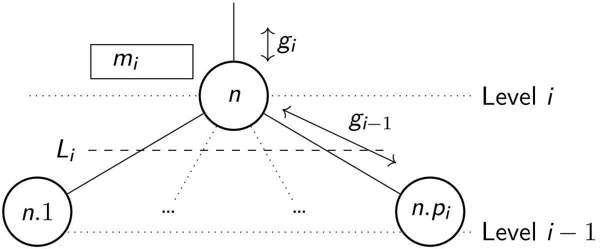

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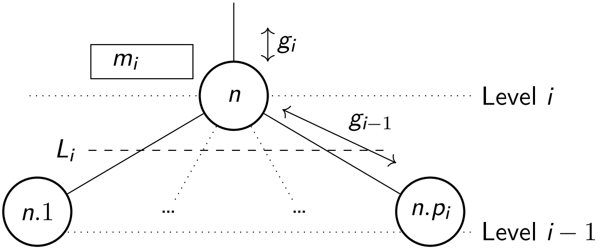

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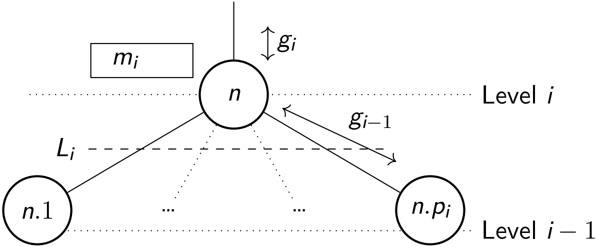
Execution model

A level i superstep is:

- Level $i-1$ executes code independantly
- Exchanges informations with the m_i memory
- Synchronises



The MULTI-BSP model

Cost model

- L : Tree levels
- N : Supersteps
- $h_{k,i}$: Max of h-relations within the i^{th} superstep at level k
- $w_{k,i}$: Max of work within the i^{th} superstep at level k

MULTI-BSP cost

$$\sum_{k=0}^{L-1} (\sum_{i=0}^{N_k-1} w_{k,i} + h_{k,i} g_k + l_k)$$

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MULTI-ML

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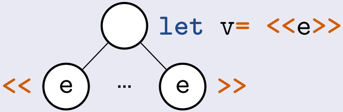
- *Multi-functions* that recursively go through the tree.



MULTI-ML: Tree recursion

Recursion structure

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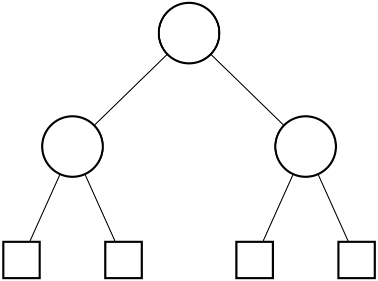

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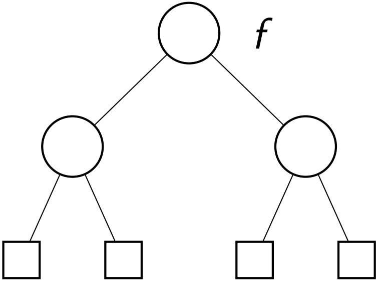

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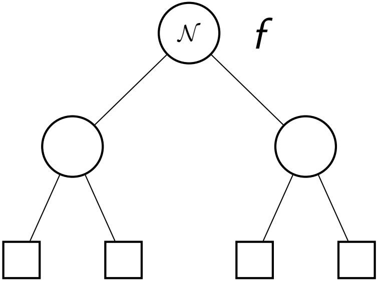

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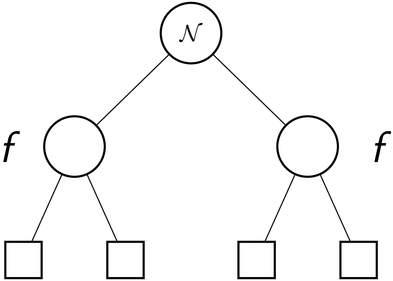

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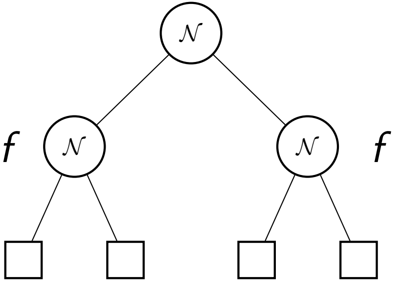

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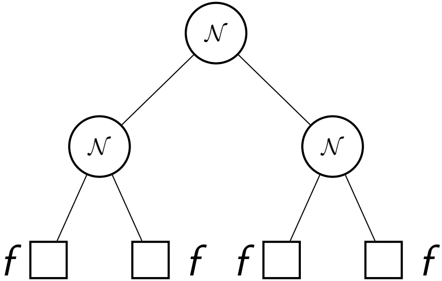

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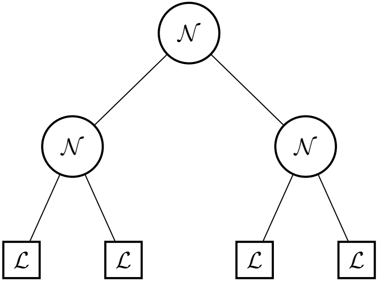

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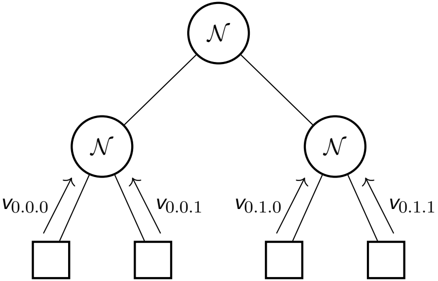

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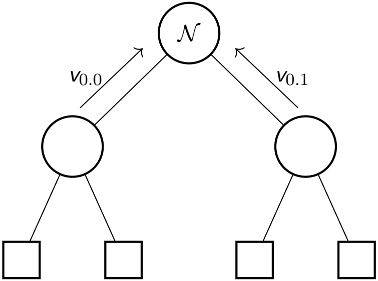

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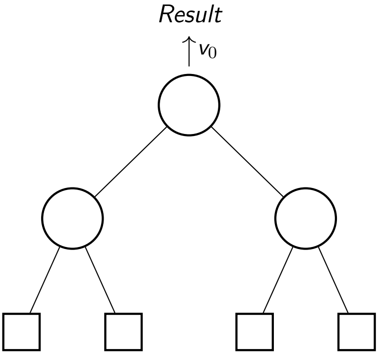

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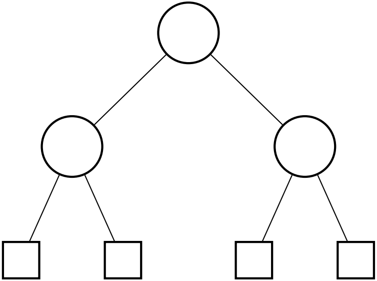

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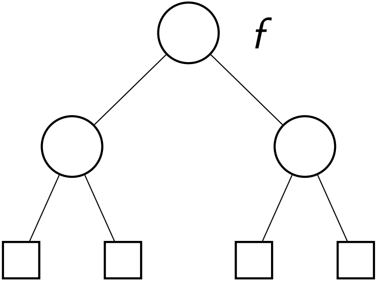

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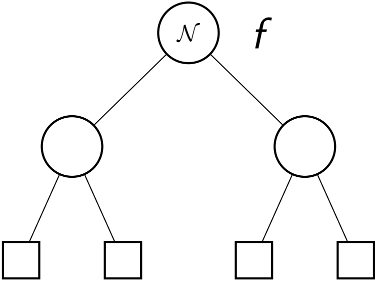

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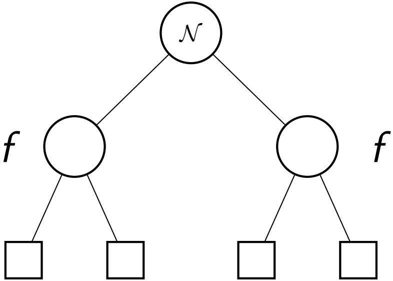

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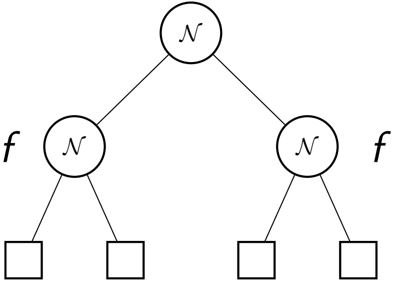

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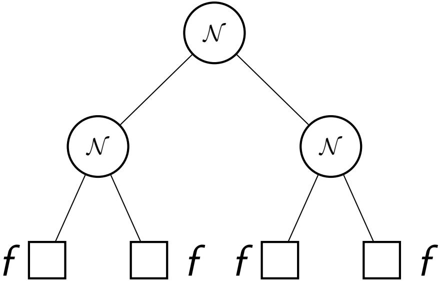

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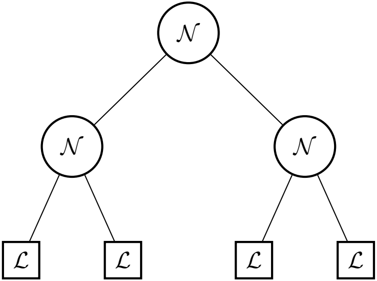

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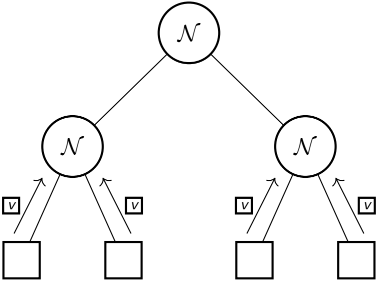

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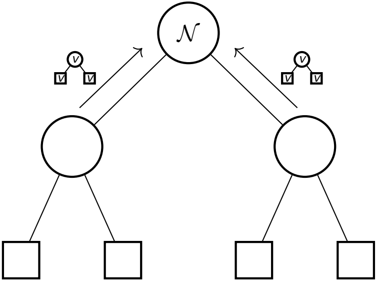

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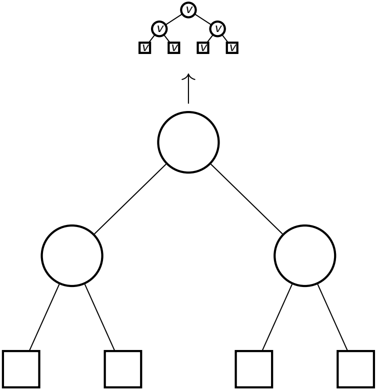

Primitives

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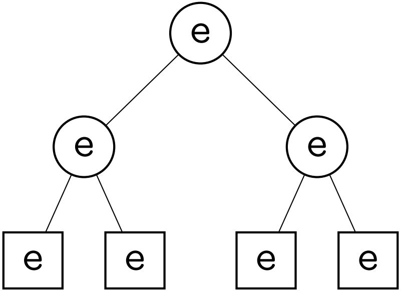

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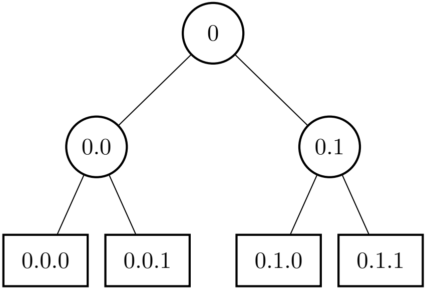

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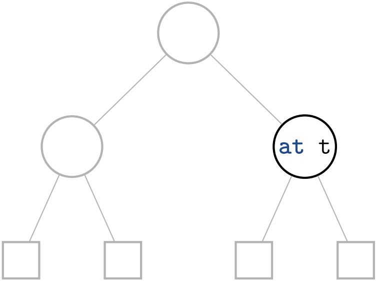

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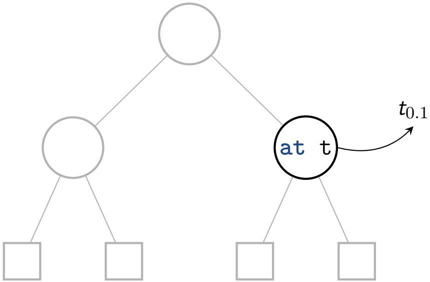

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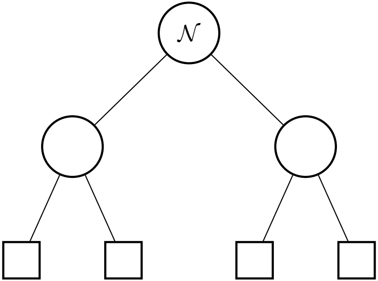

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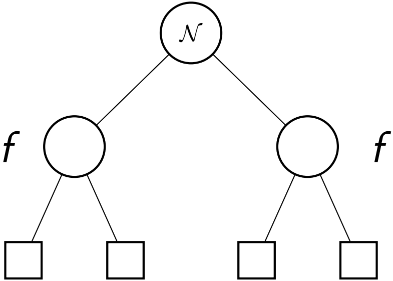

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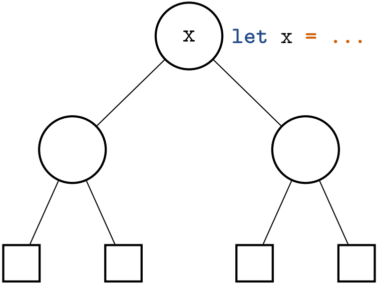

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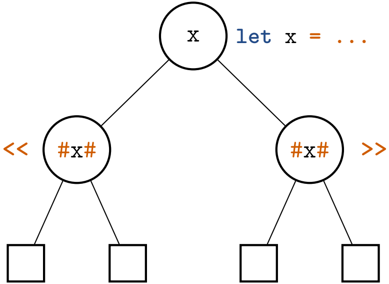

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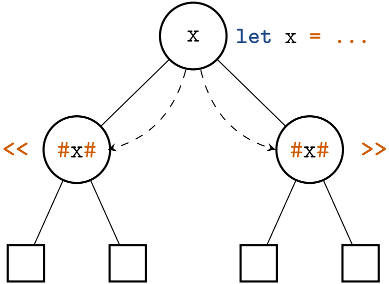

Primitives

Summary

- `mktree e`
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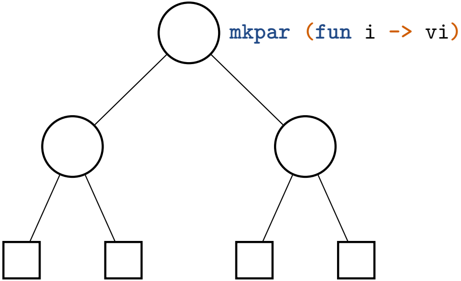

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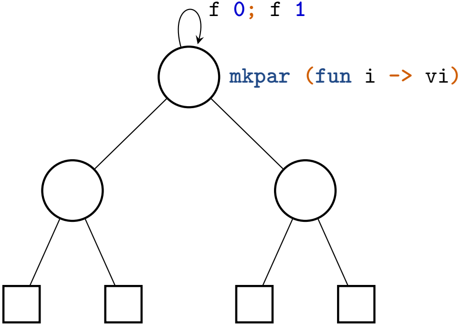

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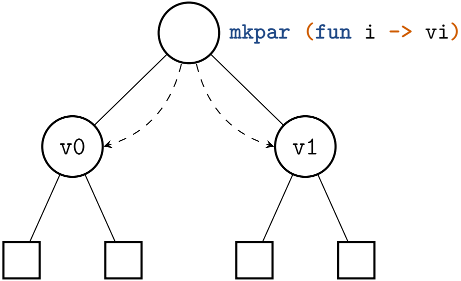
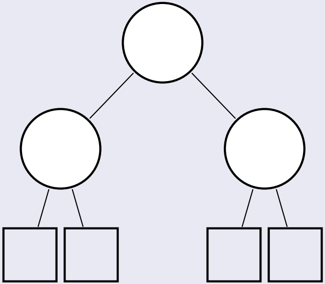


Code example

Keep the intermediate results of the sum



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let multi tree sum_list l =
  where node =
    let v = mkpar (fun i -> split i l) in
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    in (rc,s)
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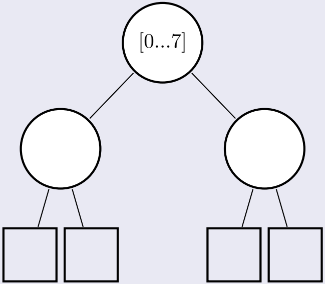

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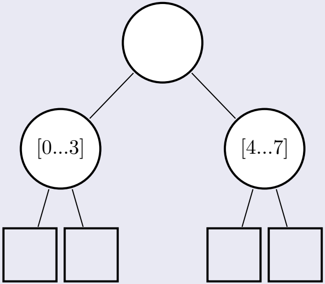

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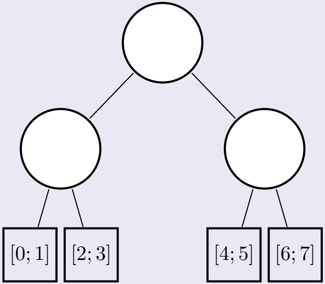

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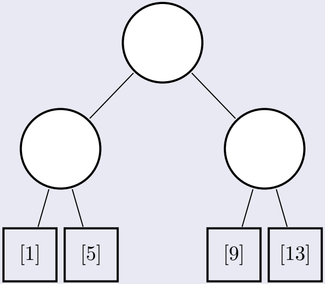

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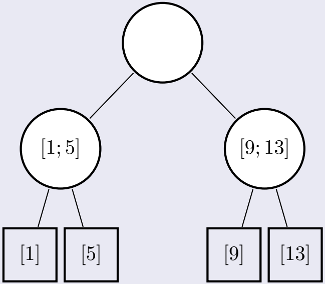

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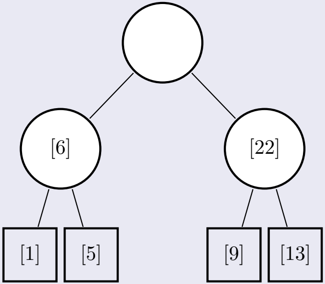

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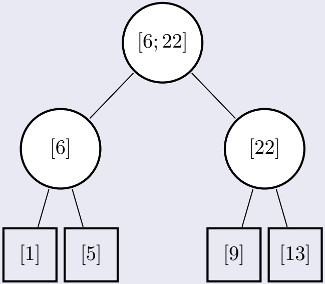

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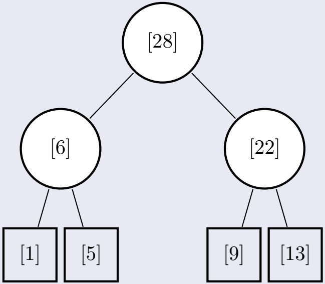

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Semantics

Formal definition of a core-languge

Useful for:

- Study of properties
- Proof of programs/compiler/typing rules

Currently

- Inductive big-step: confluent
- Co-inductive: mutually exclusive

Typing

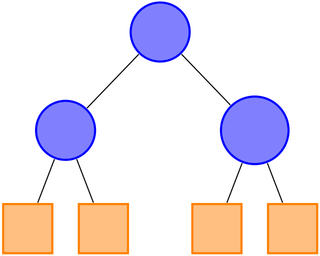
Purely **C**onstraint-**B**ased system : **PCB(X)**

- Constraint based
- Extension of DM's type system
- Easy to extend
- Related to HM(X)

MULTI-ML type extension

- Add parallel constructions
- Introduce localities using effects (s, ℓ, b and m)
- Control parallel structure imbrications

Type localities



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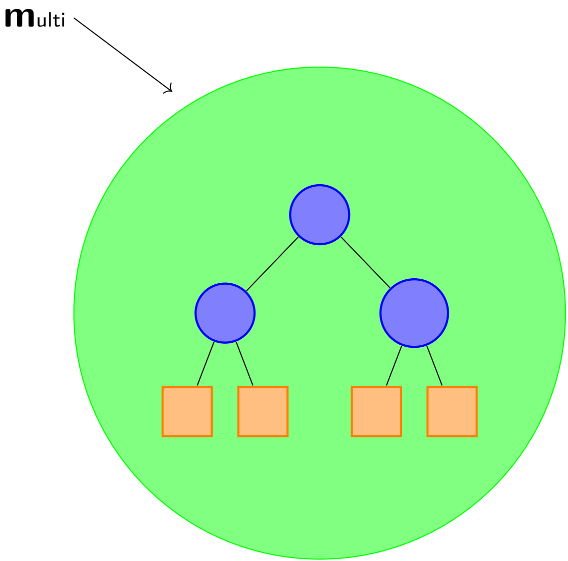

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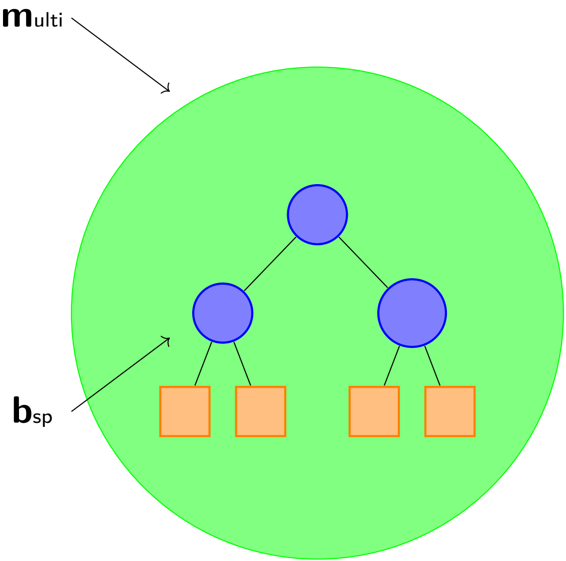

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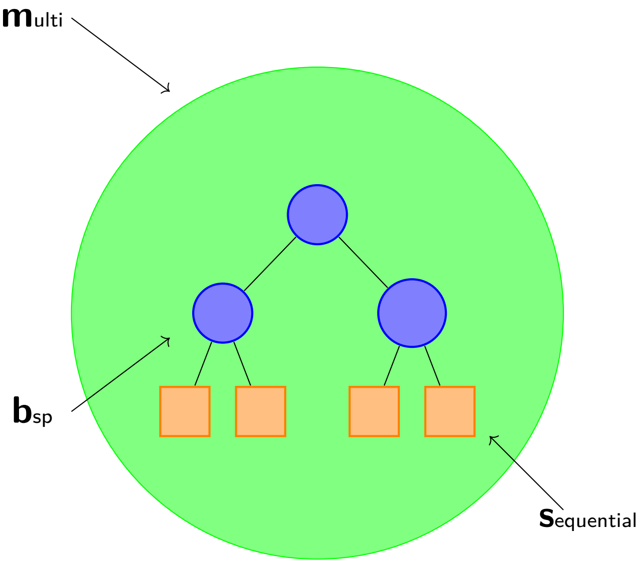

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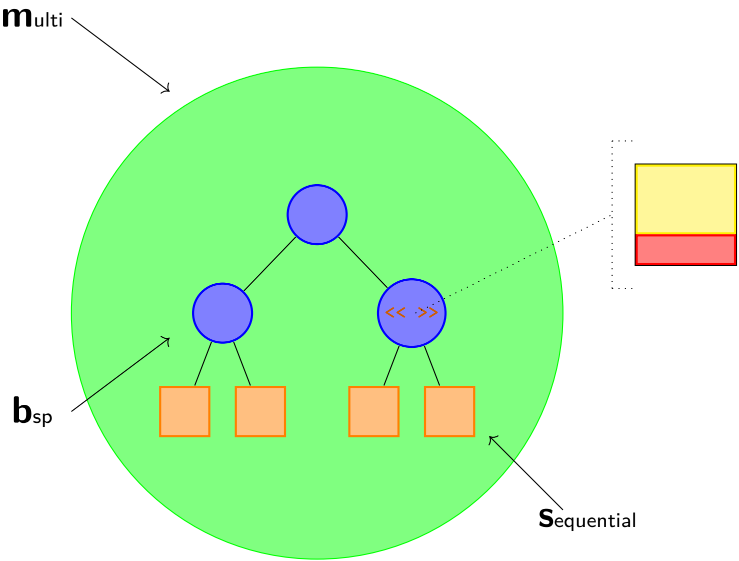

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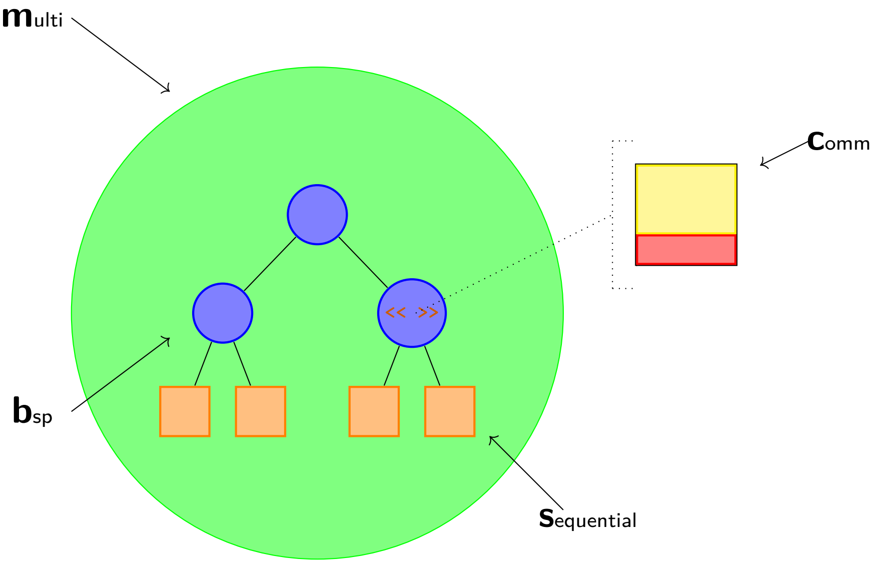

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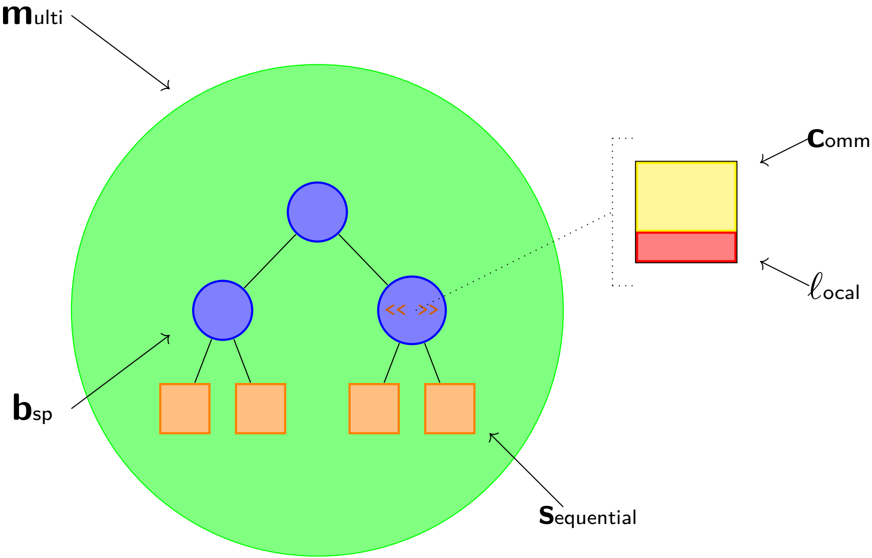

multi

comm

ℓocal

<< >>

bsp

sequential

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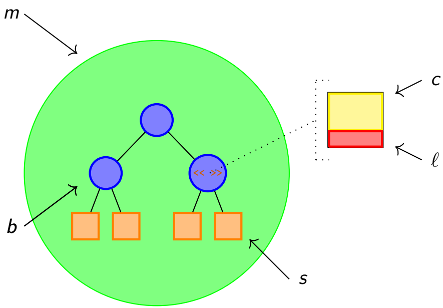

Accessibility: ◁

$$\begin{tikzcd}
 & & m & & \\
b \arrow[urr] & c \arrow[ur] \arrow[rr, leftrightarrow] & & \ell \arrow[ul] & s \arrow[ull]
\end{tikzcd}$$

Definability: ◀

$$\begin{tikzcd}
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\end{tikzcd}$$

Type syntax

Tagged type

τ	$::=$	α_π	*type variable*
		$\texttt{Base}_\pi$	*base type*
		$(\tau \xrightarrow{\pi} \tau)_\pi$	*arrow type*
		$(\tau, \tau)_\pi$	*pairs*
		$\tau\ \texttt{Par}_b$	*parallel vector*
		$\tau\ \texttt{Tree}_\pi$	*tree*

Latent effect

$\mathsf{f} : (\texttt{int}`_a \xrightarrow{b} \texttt{int}_c\, \texttt{par}_b)_m$

Implementation

Sequential simulator

- OCAML-like toplevel
- Test and debug
- Tree sturcture
- Hash tables to represent memories

```
#let multi tree f n =
  where node =
     let r =<<f ($pid$ + #n# + 1) >> in
       (r,(gid^"=>"^n))
  where leaf=
     (gid^"=>"^n);;

- : val f : int -> string tree = <multi-fun>
# (f 0)
o "0->0"
|
--o "0.0->1"
| |--> "0.0.0-> 2"
| |--> "0.0.1-> 3"
--o "0.1->2"
| |--> "0.1.0-> 3"
| |--> "0.1.1-> 4"
```

Distributed implementation

Our approach

- Modular
- Generic functors
- Communication routines
- Portable on shared and distributed memories

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- Modular
- Generic functors
- Communication routines
- Portable on shared and distributed memories

Current version

- Based on MPI
- SPMD
- One process for each nodes/leaves
- Distributed over physical cores
- Shared/Distributed memory optimisations

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3. **Results**
4. Conclusion

Benchmarks

Naive Eratosthenes algorithm

- $\sqrt(n)$th first prime numbers
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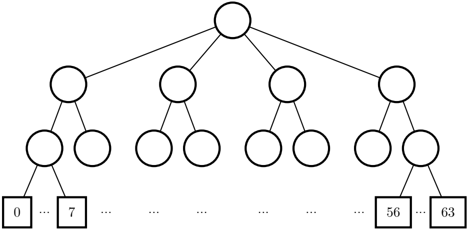

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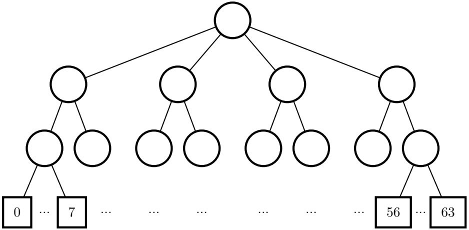

Results

	100_000		500_000		1_000_000	
	MULTI-ML	BSML	MULTI-ML	BSML	MULTI-ML	BSML
8	0.7	1.8	22.4	105.0	125.3	430.7
64	0.3	0.3	1.3	8.7	4.1	56.1
128	0.5	0.45	2.1	5.2	4.7	24.3

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Current/Future work

- Optimise MPI implementation
- Type system for MULTI-ML
- Real life benchmarks

Merci !

Any questions ?