# Multi-ML: Programming Multi-BSP Algorithms in ML

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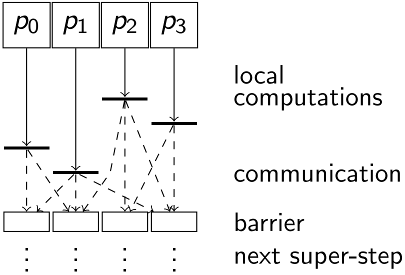

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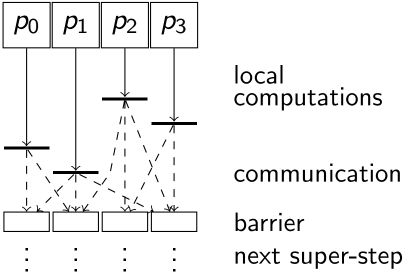

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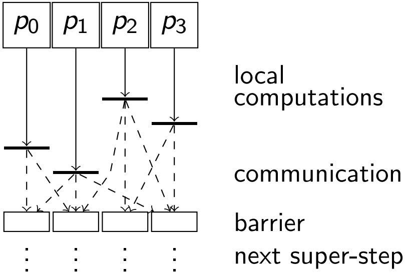

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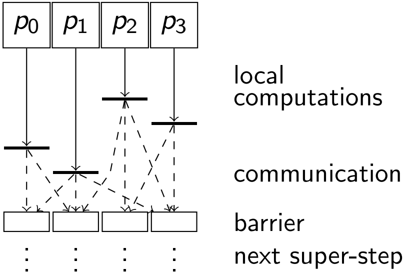
- Predictable performances



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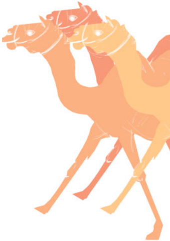
- Explicit BSP programming with a functional approach



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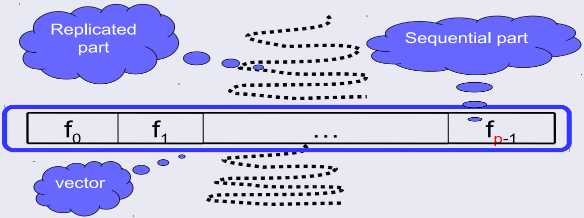
## Main idea

Parallel data structure ⇒ vectors:



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- \$**pid**\$ : A predefined vector: $i$ on processor $i$

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## Synchronous primitives

- **proj** : $\langle x_0, \ldots, x_{\mathbf{p}-1} \rangle \mapsto (\mathbf{fun}\ i \rightarrow x_i)$

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## Synchronous primitives

- **proj** : $\langle x_0, \ldots, x_{\mathbf{p}-1} \rangle \mapsto (\textbf{fun } i \to x_i)$
- **put** : $\langle f_0, \ldots, f_{\mathbf{p}-1} \rangle \mapsto \langle (\textbf{fun } i \to f_i\ 0), \ldots, (\textbf{fun } i \to f_i\ (\mathbf{p}-1)) \rangle$

## Code example

For a BSP machine with 3 processors:

```
# let vec = ≪ "HLPP␣" ≫ ;;
val vec : string par = <"HLPP␣", "HLPP␣", "HLPP␣">
# let vec2 = ≪ $vec$^(string_of_int $pid$) ≫ ;;
val vec2 : string par = <"HLPP␣0", "HLPP␣1", "HLPP␣2">
# let totex v = List.map (proj v) procs;;
val totex : 'a Bsml.par → 'a list = <fun>
# totex vec2;;
— : string list = ["HLPP0"; "HLPP1"; "HLPP2"]
```

## What is MULTI-BSP?

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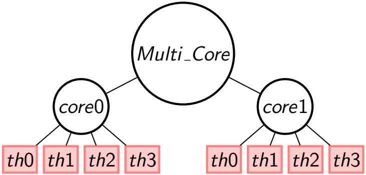

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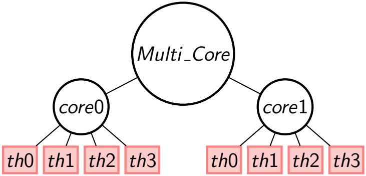
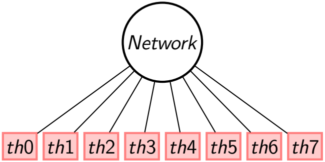
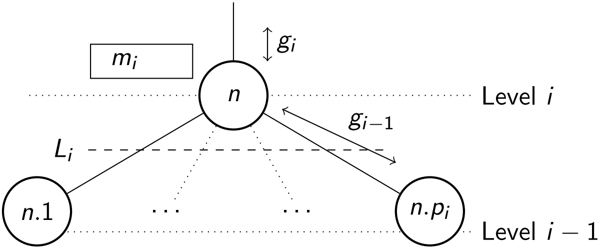



BSP



## Execution model

A level $i$ superstep is:



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A level $i$ superstep is:

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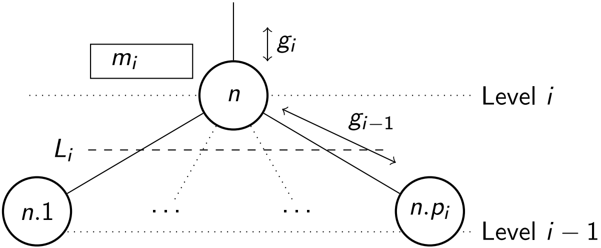

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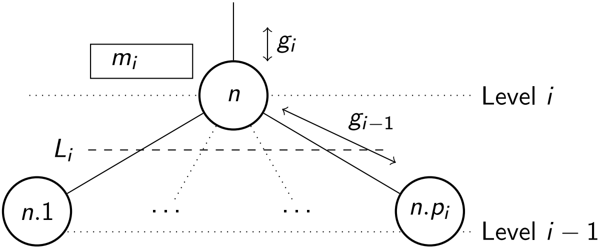
A level $i$ superstep is:

- Level $i - 1$ executes code independantly
- Exchanges informations with the $m_i$ memory



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A level $i$ superstep is:

- Level $i-1$ executes code independantly
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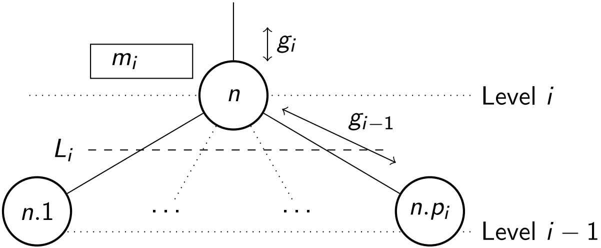
- Synchronises



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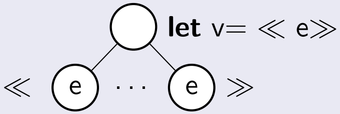

### Recursion structure

**let multi** f [args] =
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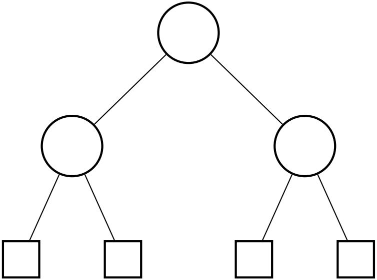

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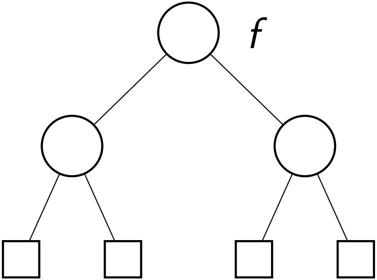

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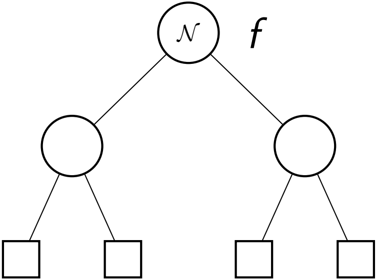

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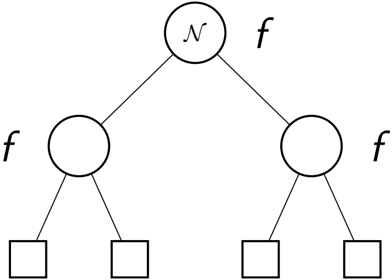

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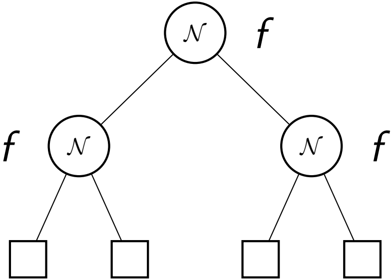

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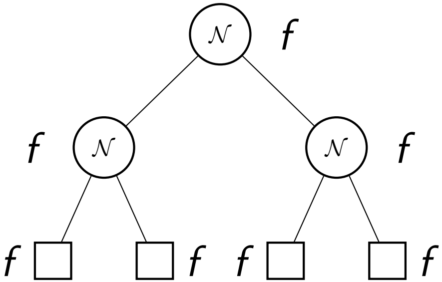

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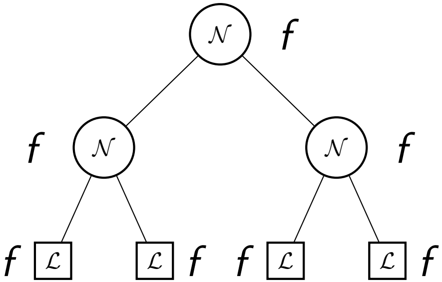

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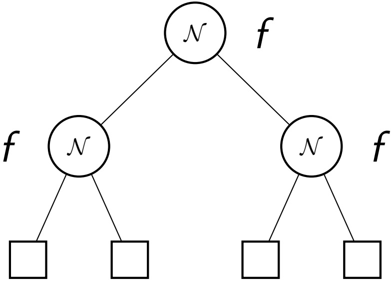

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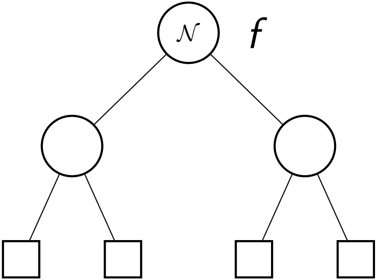

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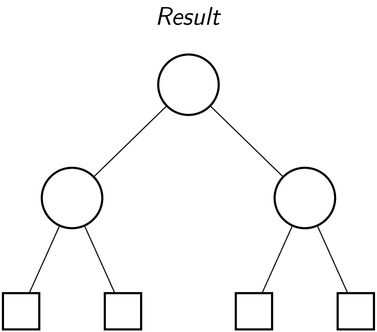

# Primitives

Summary:

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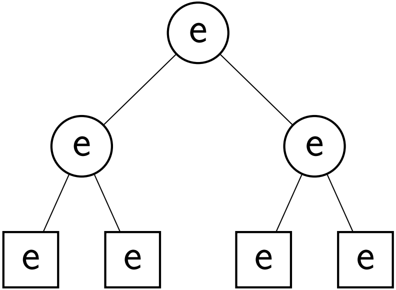
- §e§



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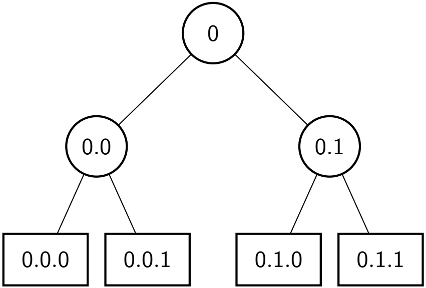

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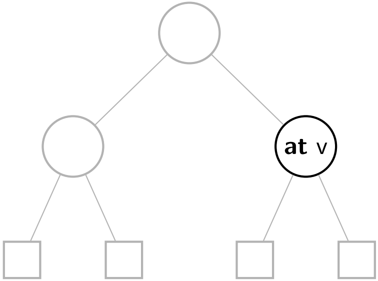

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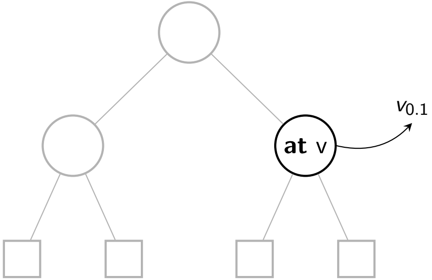

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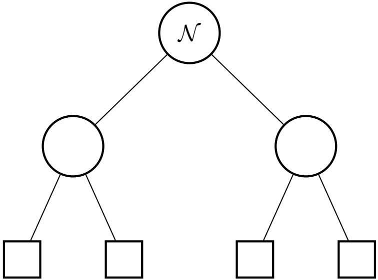

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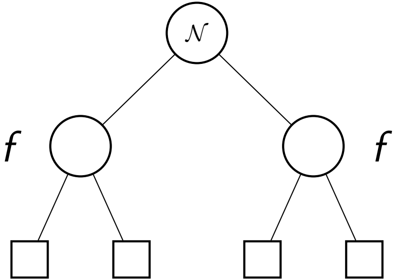

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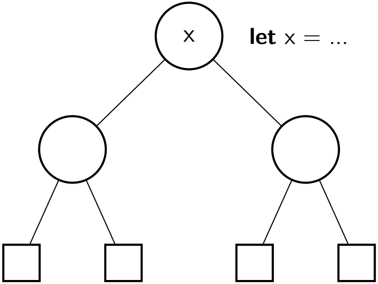

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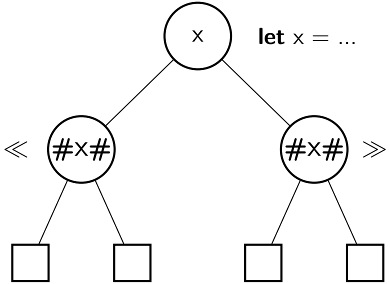

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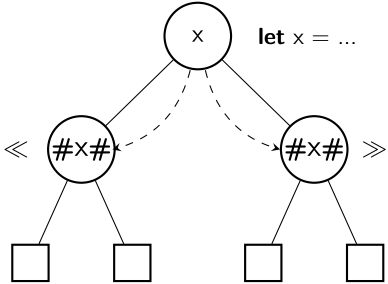

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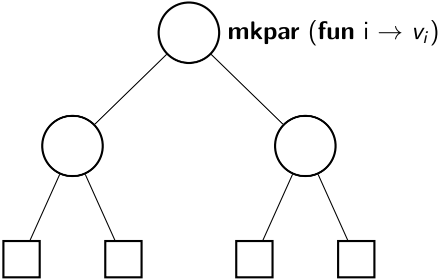
- **mkpar** f



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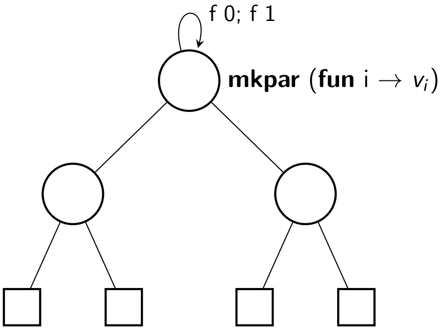

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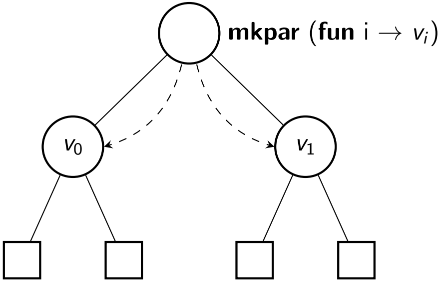

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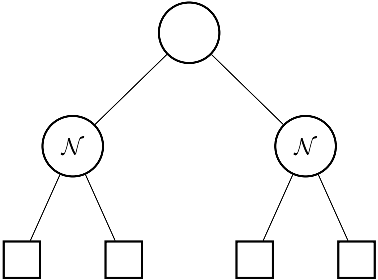

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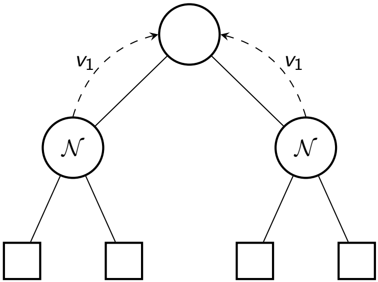

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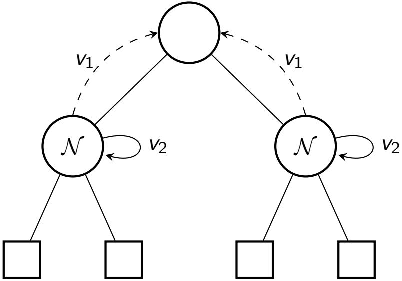

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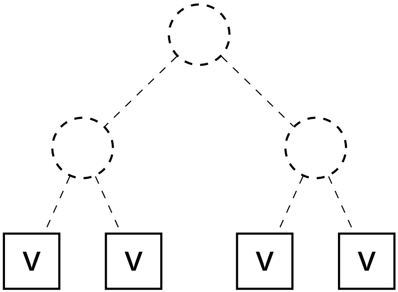
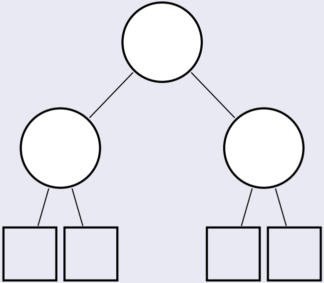


# Code example

## Keep the intermediate results of the sum:



```
let multi tree sum_list l =
   where node =
      let v = mkpar (fun i → split i l) in
      let s = sumSeq (flatten ≪ sum_list $v$≫ ) in
      finally ˜up:s ˜keep:s
   where leaf =
      let s = sumSeq l in

      finally ˜up:s ˜keep:s
```

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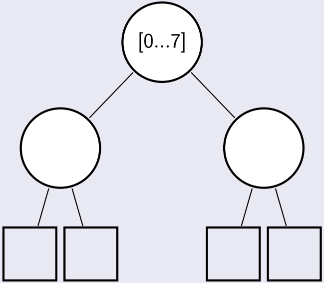


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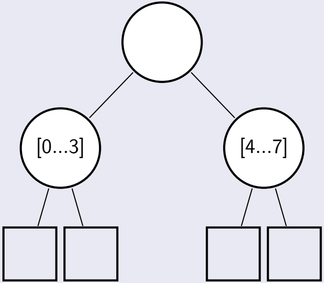
# Code example

## Keep the intermediate results of the sum:




```
let multi tree sum_list l =
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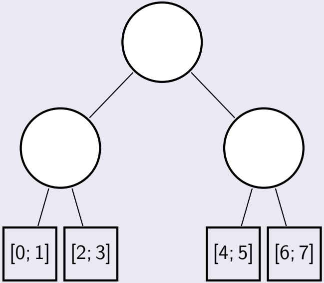


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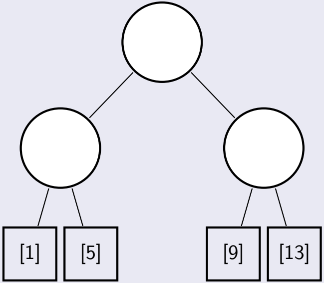


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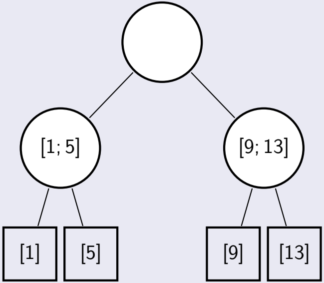


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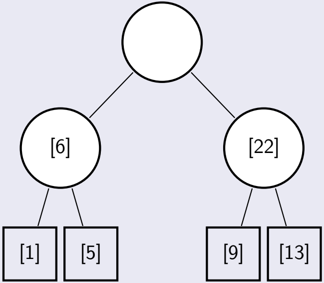


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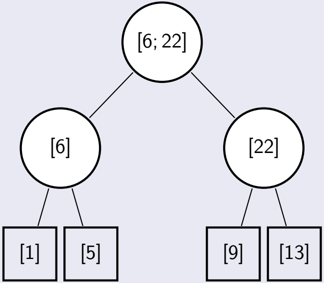


**let multi tree** sum_list l =
  **where node** =
    **let** v = **mkpar** (**fun** i → split i l) **in**
    **let** s = sumSeq (flatten ≪ sum_list $v$≫ ) **in**
    **finally** ˜**up**:s ˜**keep**:s
  **where leaf** =
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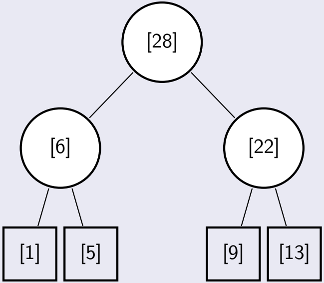


```
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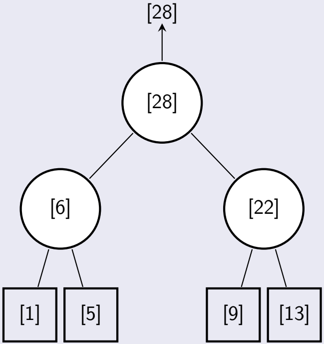


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## Formal definition of a core-languge

Useful for:

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## Currently

- Inductive big-step: confluent
- Co-inductive: mutually exclusive

# Implementation

## Sequential simulator

- OCAML-like toplevel
- Test and debug
- Tree sturcture
- Hash tables to represent memories

```
#let multi f n =
 | where node =
 | | let _=≪f ($pid$ + #n# + 1) ≫ in
 | | | finally ~up:() ~keep:(gid^"=>"^n)
 | where leaf=finally ~up:() ~keep:(gid^"=>"^n);;

— : val f : int→string tree = <multi-fun>
#(f 0)
o "0→ 0"
|
−−o "0.0→ 1"
| | −→ "0.0.0→ 2"
| | −→ "0.0.1→ 3"
−−o "0.1→ 2"
| | −→ "0.1.0→ 3"
| | −→ "0.1.1→ 4"
```

# Distributed implementation

## Our approach

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# Benchmarks

## Naive Eratosthenes algorithm

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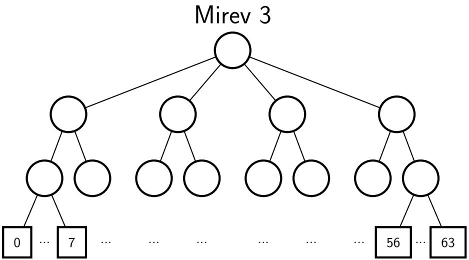

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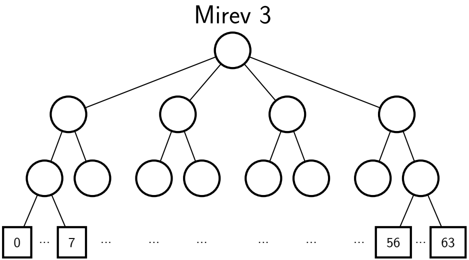


## Naive Eratosthenes algorithm

- $\sqrt(n)$th first prime numbers
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## Results

| | 100_000 | | 500_000 | | 1_000_000 | |
|---|---|---|---|---|---|---|
| | MULTI-ML | BSML | MULTI-ML | BSML | MULTI-ML | BSML |
| 8 | 0.7 | 1.8 | 22.4 | 105.0 | 125.3 | 430.7 |
| 64 | 0.3 | 0.3 | 1.3 | 8.7 | 4.1 | 56.1 |
| 128 | 0.5 | 0.45 | 2.1 | 5.2 | 4.7 | 24.3 |

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# Thank you for your attention !

Any questions ?