

# MULTI-ML: PROGRAMMING MULTI-BSP ALGORITHMS IN ML

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# Table of Contents

## ① Introduction

OCAML

BSML

MULTI-BSP

## ② Multi-ML

## ③ Results

## ④ Conclusion

# Ocaml : a ML language



## Strengths of Ocaml

- A fonctionnal programming language

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## Strengths of Ocaml

- A fonctionnal programming language
- A powerful type system
- User-definable algebraic data types and pattern matching
- Automatic memory management
- Efficient native code compilers



## Syntaxe overview

```
# let f = fun x → "Hello_"^(string_of_int x) in
  let lst = [0;1;2;3] in
    List.map f lst;;
- : string list = ["Hello_0"; "Hello_1"; "Hello_2"; "Hello_3"]
```

```
# let pair = ([0;1;2;3],true);;
val pair : int list * bool = ([0; 1; 2; 3], true)
```

```
# type 'a list =
  Nil
  | Node of 'a*'a list ;;
type 'a list = Nil | Node of 'a * 'a list
```

# Bulk Synchronous ML

What is BSML?



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## What is BSML?

- Explicit BSP programming with a functional approach



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- Formal semantics  $\rightarrow$  computer-assisted proofs (COQ)



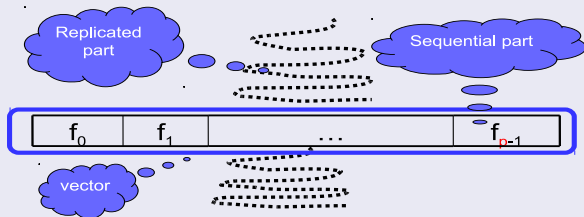
# Bulk Synchronous ML

## What is BSML?

- Explicit BSP programming with a functional approach
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## Main idea

Parallel data structure  $\Rightarrow$  vectors:



# BSML primitives

## Asynchronous primitives

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### Synchronous primitives

- **proj** :  $\langle x_0, \dots, x_{p-1} \rangle \mapsto (\text{fun } i \rightarrow x_i)$
- **put** :  $\langle f_0, \dots, f_{p-1} \rangle \mapsto \langle (\text{fun } i \rightarrow f_i \ 0), \dots, (\text{fun } i \rightarrow f_i \ (p-1)) \rangle$

## Code example

For a BSP machine with 3 processors:

```
# let vec = << "Hello_" >> ;;  
val vec : string par = <"Hello_", "Hello_", "Hello_">  
# let vec2 = << $vec$^(string_of_int $pid$) >> ;;  
val vec2 : string par = <"Hello_0", "Hello_1", "Hello_2">  
# let totex v = List.map (proj v) procs;;  
val totex : 'a Bsml.par → 'a list = <fun>  
# totex vec2;;  
— : string list = ["Hello0"; "Hello1"; "Hello2"]
```

# The MULTI-BSP model

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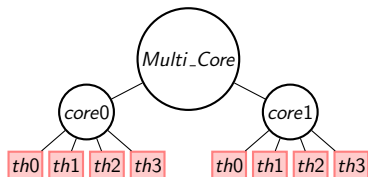
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# The MULTI-BSP model

## What is MULTI-BSP?

- 1 A tree structure with nested components
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- 3 And leaves are processors

### MULTI-BSP

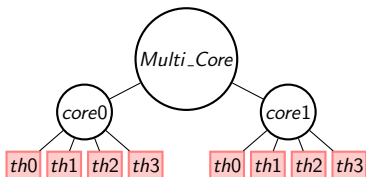


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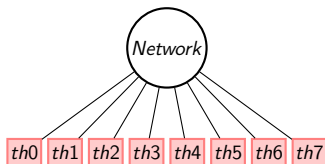
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MULTI-BSP



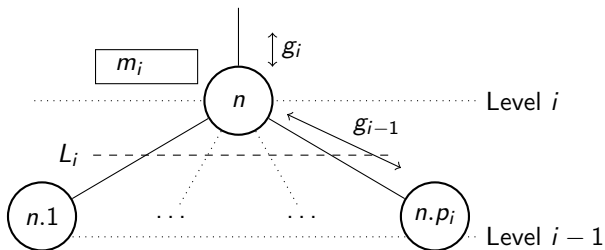
BSP



# The MULTI-BSP model

## Execution model

A level  $i$  superstep is:

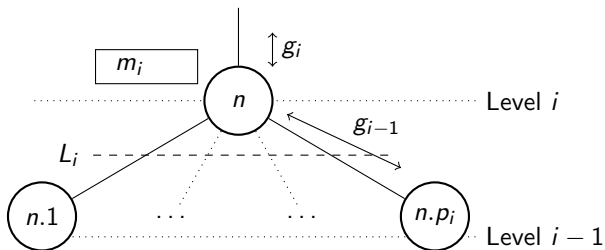


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## Execution model

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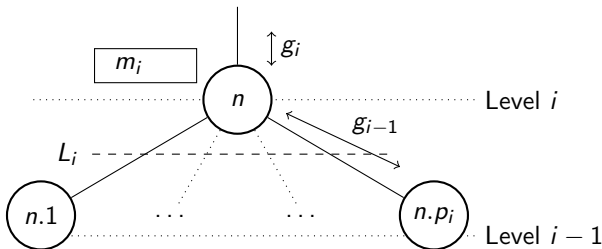


# The MULTI-BSP model

## Execution model

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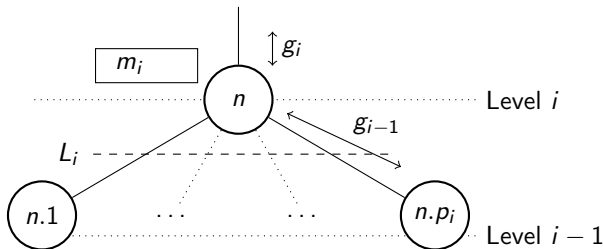


# The MULTI-BSP model

## Execution model

A level  $i$  superstep is:

- Level  $i - 1$  executes code independantly
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- Synchronises



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## 2 Multi-ML

- Overview

- Primitives

- Semantics

- Typing

- Implementation

## 3 Results

## 4 Conclusion



Basic ideas:

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- BSML-like code on every stage of the MULTI-BSP architecture

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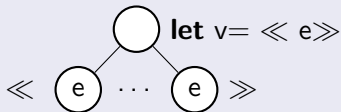
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## MULTI-ML: Tree recursion

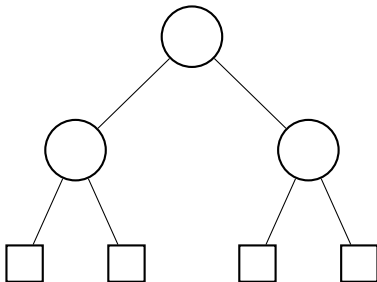
### Recursion structure

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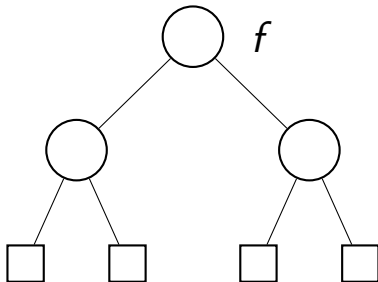
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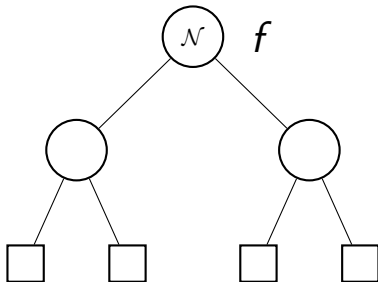




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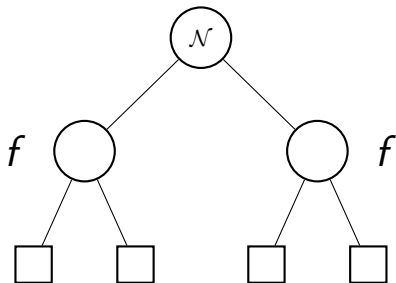
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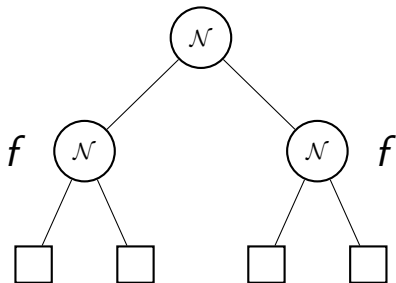
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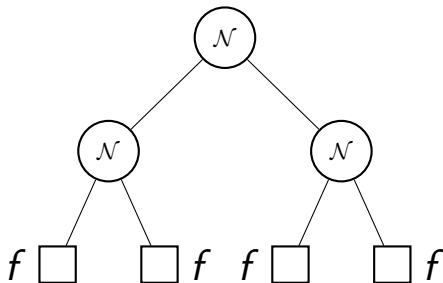
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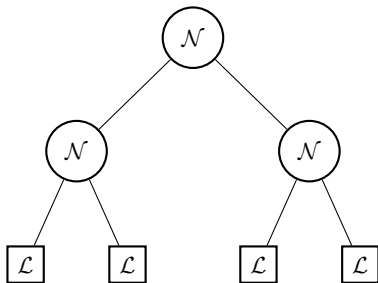
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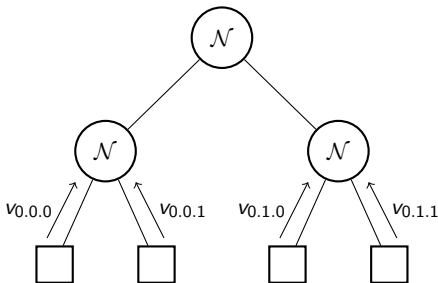
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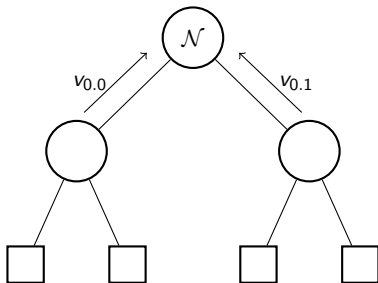
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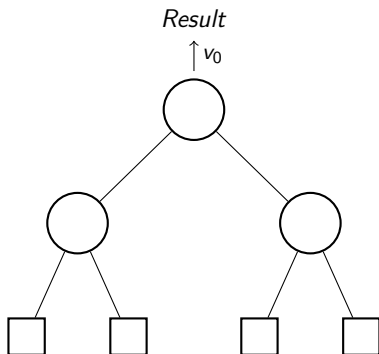
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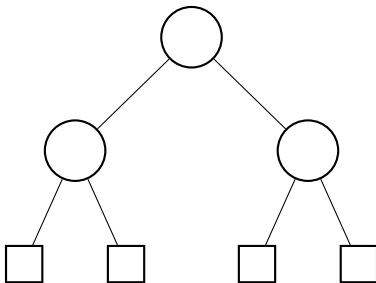
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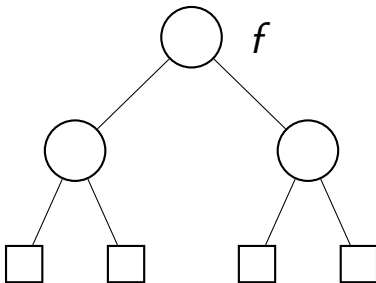
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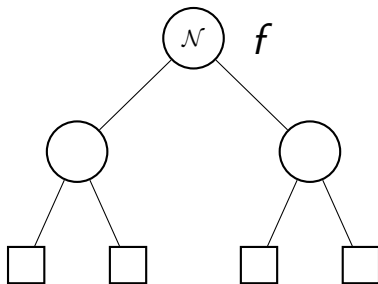
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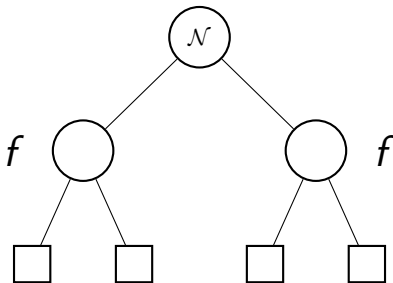
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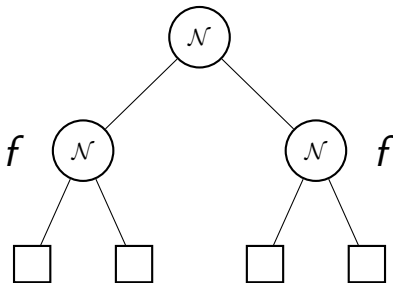
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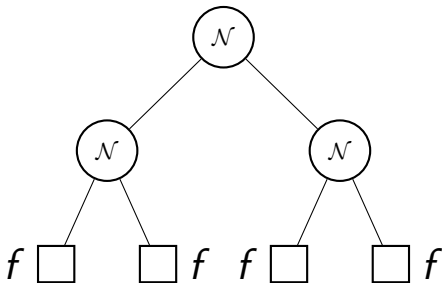
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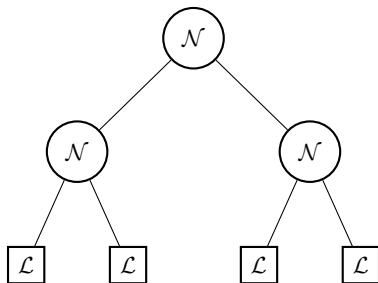
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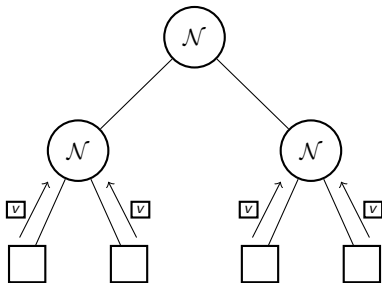




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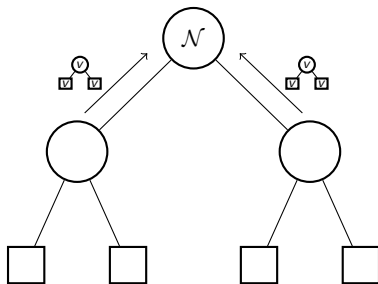
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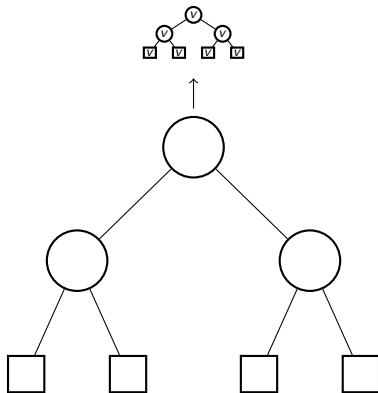
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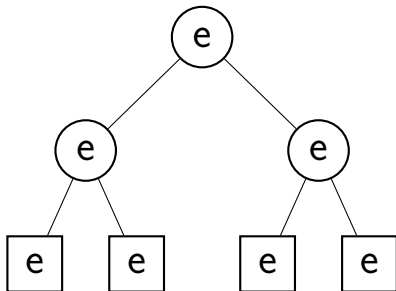
# Primitives

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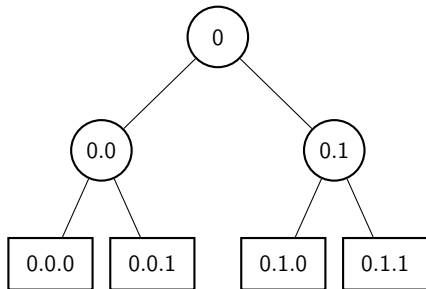
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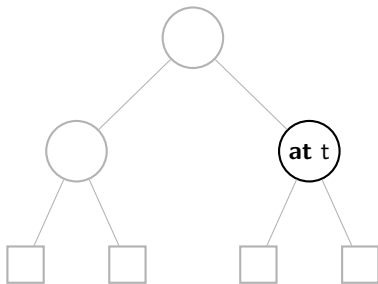
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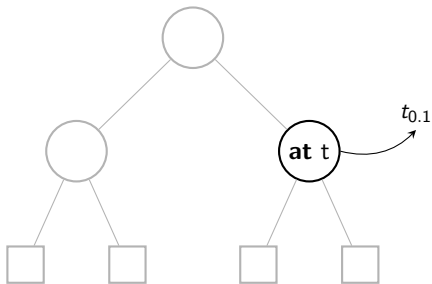
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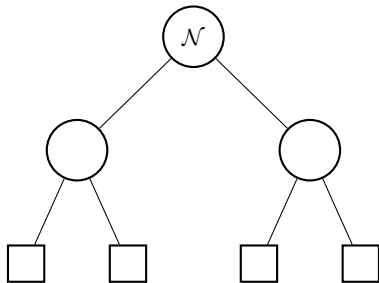




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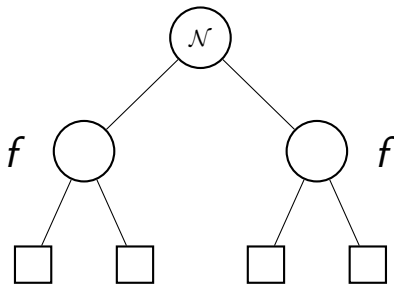
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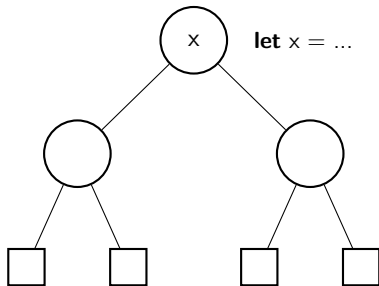
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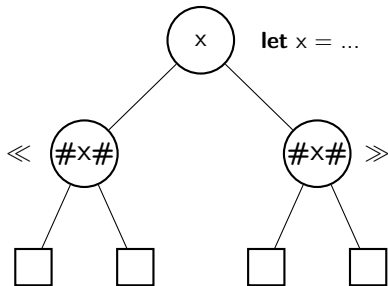
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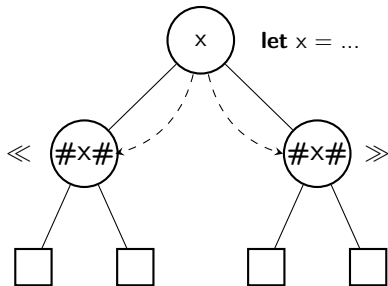
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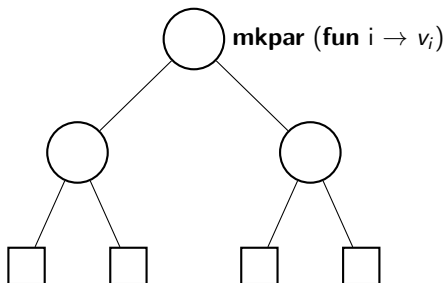
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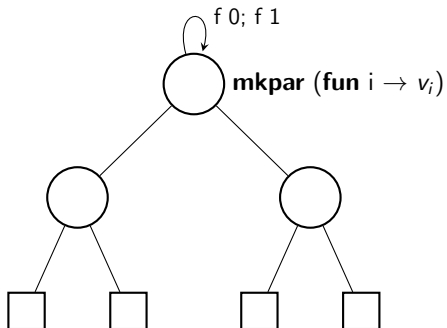
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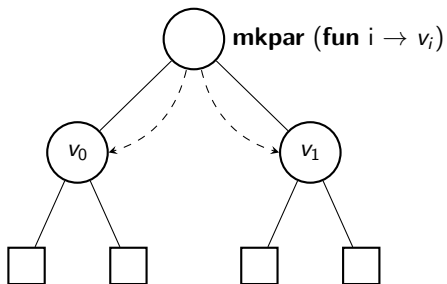
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- **mkpar**  $f$



# Primitives

## Summary:

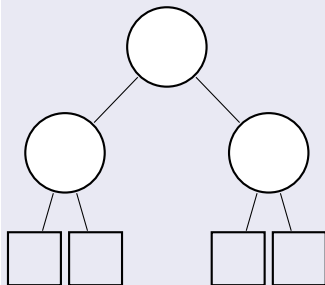
- $\S e \S$
- **gid**
- **at**
- $\ll \dots f \dots \gg$
- $\#x\#$
- **mkpar**  $f$





## Code example

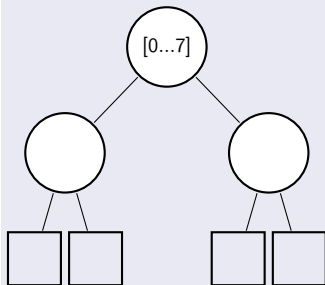
Keep the intermediate results of the sum:



```
let multi tree sum_list l =  
  where node =  
    let v = mkpar (fun i → split i l) in  
    let rc = << sum_list $v$>> in  
    let s = sumSeq (flatten << at $rc$>> ) in  
    (rc,s)  
  where leaf =  
    sumSeq l
```

## Code example

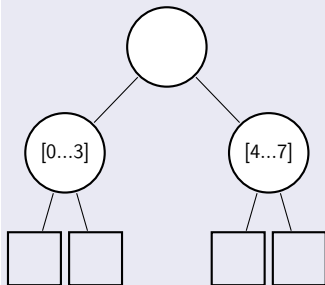
Keep the intermediate results of the sum:



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## Code example

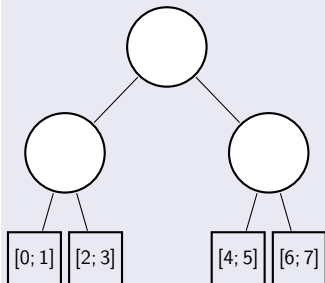
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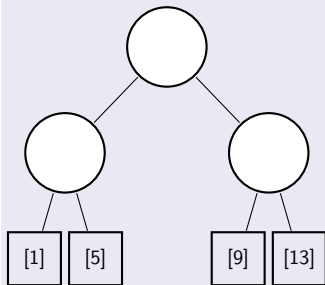
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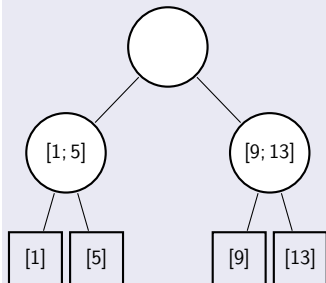
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## Code example

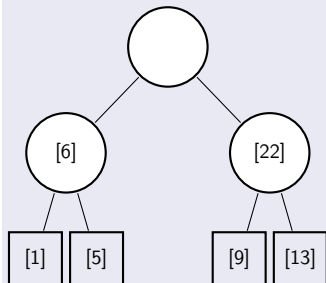
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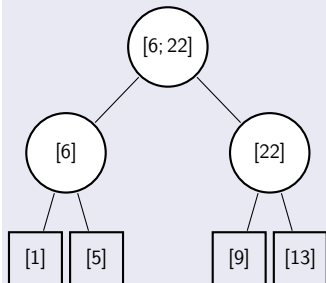
Keep the intermediate results of the sum:



```
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```

## Code example

Keep the intermediate results of the sum:

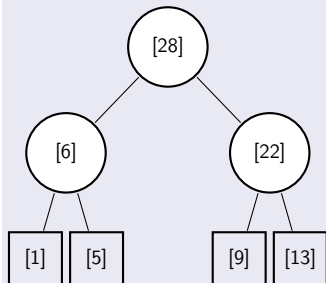


```
let multi tree sum_list l =  
  where node =  
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## Code example

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```

## Formal definition of a core-language

Useful for:

- Study of properties
- Proof of programs/compiler/typing rules

## Currently

- Inductive big-step: confluent
- Co-inductive: mutually exclusive

## Purely Constraint-Based system : **PCB(X)**

- Constraint based
- Extension of DM's type system
- Easy to extend
- Related to HM(X)

## MULTI-ML type extension

- Add parallel constructions
- Introduce locality ( $s, \ell, b$  and  $m$ ) using effects
- Reject nested vectors
- Consistency

# Implementation

## Sequential simulator

- OCAML-like toplevel
- Test and debug
- Tree structure
- Hash tables to represent memories

```
#let multi tree f n =  
  where node =  
    | let r = <<f ($pid$ + #n# + 1) >> in  
    | (r, (gid^"=>"^n))  
  where leaf =  
    | (gid^"=>"^n);;
```

```
— : val f : int→string tree = <multi-fun>  
#(f 0)  
o "0→ 0"  
|  
--o "0.0→ 1"  
| | --> "0.0.0→ 2"  
| | --> "0.0.1→ 3"  
--o "0.1→ 2"  
| | --> "0.1.0→ 3"  
| | --> "0.1.1→ 4"
```

# Distributed implementation

## Our approach

# Distributed implementation

## Our approach

- Modular

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- Generic functors

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- Portable on shared and distributed memories

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- Based on MPI

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- Based on MPI
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- Distributed over physical cores

# Distributed implementation

## Our approach

- Modular
- Generic functors
- Communication routines
- Portable on shared and distributed memories

## Current version

- Based on MPI
- SPMD
- One process for each nodes/leaves
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- Shared/Distributed memory optimisations

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### Naive Eratosthenes algorithm

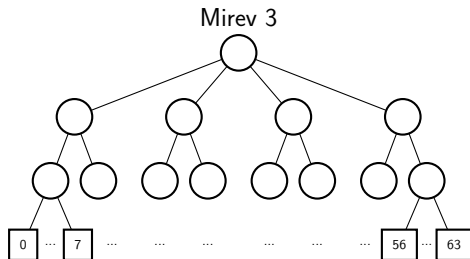
- $\sqrt{n}$ th first prime numbers
- Based on scan
- Unbalanced



## Benchmarks

### Naive Eratosthenes algorithm

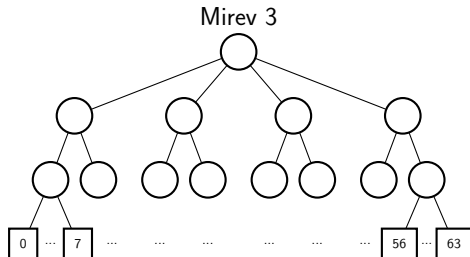
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## Benchmarks

### Naive Eratosthenes algorithm

- $\sqrt{(n)}$ th first prime numbers
- Based on scan
- Unbalanced



## Results

	100_000		500_000		1_000_000	
	MULTI-ML	BSML	MULTI-ML	BSML	MULTI-ML	BSML
8	0.7	1.8	22.4	105.0	125.3	430.7
64	0.3	0.3	1.3	8.7	4.1	56.1
128	0.5	0.45	2.1	5.2	4.7	24.3

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# Conclusion

MULTI-ML

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## Current/Future work

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## Current/Future work

- Optimise MPI implementation
- Type system for MULTI-ML
- Real life benchmarks

Merci !

Any questions ?