1. Langage WHILE

Grammaire des expressions et des valeurs :

$$\begin{array}{rcl} e & ::= & \bar{n} \mid e+e \mid e\times e \mid e-e \mid e=e \mid e>e \mid e<e \\ & \mid & x \mid \textbf{true} \mid \textbf{false} \mid e \textbf{ and } e \mid e \textbf{ or } e \mid \textbf{not } e \end{array}$$

Grammaire des types :

$$\sigma,\tau ::= \textbf{int} \mid \textbf{bool}$$

Typage :

$$\frac{}{\Gamma\vdash \bar{n}:\textbf{int}} \qquad \frac{}{\Gamma\vdash \textbf{true}:\textbf{int}} \qquad \frac{}{\Gamma\vdash \textbf{false}:\textbf{bool}}$$

$$\frac{\Gamma\vdash e_1:\textbf{int} \quad \Gamma\vdash e_2:\textbf{int}}{\Gamma\vdash e_1+e_2:\textbf{int}} \qquad \frac{\Gamma\vdash e_1:\textbf{int} \quad \Gamma\vdash e_2:\textbf{int}}{\Gamma\vdash e_1=e_2:\textbf{bool}} \qquad \frac{\Gamma\vdash e_1:\textbf{bool} \quad \Gamma\vdash e_2:\textbf{bool}}{\Gamma\vdash e_1 \textbf{ and } e_2:\textbf{bool}}$$

$$\frac{\Gamma\vdash e_1:\textbf{int} \quad \Gamma\vdash e_2:\textbf{int}}{\Gamma\vdash e_1-e_2:\textbf{int}} \qquad \frac{\Gamma\vdash e_1:\textbf{int} \quad \Gamma\vdash e_2:\textbf{int}}{\Gamma\vdash e_1>e_2:\textbf{bool}} \qquad \frac{\Gamma\vdash e_1:\textbf{bool} \quad \Gamma\vdash e_2:\textbf{bool}}{\Gamma\vdash e_1 \textbf{ or } e_2:\textbf{bool}}$$

$$\frac{\Gamma\vdash e_1:\textbf{int} \quad \Gamma\vdash e_2:\textbf{int}}{\Gamma\vdash e_1\times e_2:\textbf{int}} \qquad \frac{\Gamma\vdash e_1:\textbf{int} \quad \Gamma\vdash e_2:\textbf{int}}{\Gamma\vdash e_1<e_2:\textbf{bool}} \qquad \frac{\Gamma\vdash e:\textbf{bool}}{\Gamma\vdash \textbf{not } e:\textbf{bool}}$$

$$\frac{x:\tau\in\Gamma}{\Gamma\vdash x:\tau}$$

Sémantique naturelle :

$$\frac{}{\mu\vdash \bar{n} \Downarrow \bar{n}} \qquad \frac{(x,v)\in\mu}{\mu\vdash x\Downarrow v}$$

$$\frac{}{\mu\vdash \textbf{true}\Downarrow\textbf{true}} \qquad \frac{}{\mu\vdash \textbf{false}\Downarrow\textbf{false}}$$

$$\frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m}}{\mu\vdash e_1+e_2\Downarrow\overline{n+m}} \qquad \frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m}}{\mu\vdash e_1-e_2\Downarrow\overline{n-m}}$$

$$\frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m}}{\mu\vdash e_1\times e_2\Downarrow\overline{n\times m}}$$

$$\frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m} \quad n<m}{\mu\vdash e_1<e_2\Downarrow\textbf{true}} \qquad \frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m} \quad n>m}{\mu\vdash e_1>e_2\Downarrow\textbf{true}}$$

$$\frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m} \quad n\geqslant m}{\mu\vdash e_1<e_2\Downarrow\textbf{false}} \qquad \frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m} \quad n\leqslant m}{\mu\vdash e_1>e_2\Downarrow\textbf{false}}$$

$$\frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m} \quad n=m}{\mu\vdash e_1=e_2\Downarrow\textbf{true}} \qquad \frac{\mu\vdash e_1\Downarrow\bar{n} \quad \mu\vdash e_2\Downarrow\bar{m} \quad n\neq m}{\mu\vdash e_1=e_2\Downarrow\textbf{false}}$$

$$\frac{\mu\vdash e_1\Downarrow\textbf{true}}{\mu\vdash e_1 \textbf{ or } e_2\Downarrow\textbf{true}} \qquad \frac{\mu\vdash e_1\Downarrow\textbf{false}}{\mu\vdash e_1 \textbf{ and } e_2\Downarrow\textbf{false}}$$

$$\frac{\mu\vdash e_1\Downarrow\textbf{false} \quad \mu\vdash e_2\Downarrow v}{\mu\vdash e_1 \textbf{ or } e_2\Downarrow v} \qquad \frac{\mu\vdash e_1\Downarrow\textbf{true} \quad \mu\vdash e_2\Downarrow v}{\mu\vdash e_1 \textbf{ and } e_2\Downarrow v}$$

$$\frac{\mu\vdash e\Downarrow\textbf{true}}{\mu\vdash \textbf{not } e\Downarrow\textbf{false}} \qquad \frac{\mu\vdash e\Downarrow\textbf{false}}{\mu\vdash \textbf{not } e\Downarrow\textbf{true}}$$

Sémantique opérationnelle structurée :

$$\frac{(x,v)\in\mu}{\mu\vdash x \;\to\; v}$$

$$\frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1+e_2 \;\to\; e_1'+e_2} \qquad \frac{\mu\vdash e_2 \;\to\; e_2'}{\mu\vdash \bar{n}+e_2 \;\to\; \bar{n}+e_2'} \qquad \frac{}{\mu\vdash \bar{n}+\bar{m} \;\to\; \overline{n+m}}$$

$$\frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1\times e_2 \;\to\; e_1'\times e_2} \qquad \frac{\mu\vdash e_2 \;\to\; e_2'}{\mu\vdash \bar{n}\times e_2 \;\to\; \bar{n}\times e_2'} \qquad \frac{}{\mu\vdash \bar{n}\times\bar{m} \;\to\; \overline{n\times m}}$$

$$\frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1-e_2 \;\to\; e_1'-e_2} \qquad \frac{\mu\vdash e_2 \;\to\; e_2'}{\mu\vdash \bar{n}-e_2 \;\to\; \bar{n}-e_2'} \qquad \frac{}{\mu\vdash \bar{n}-\bar{m} \;\to\; \overline{n-m}}$$

$$\frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1<e_2 \;\to\; e_1'<e_2} \qquad \frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1>e_2 \;\to\; e_1'>e_2} \qquad \frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1=e_2 \;\to\; e_1'=e_2}$$

$$\frac{\mu\vdash e_2 \;\to\; e_2'}{\mu\vdash \bar{n}<e_2 \;\to\; \bar{n}<e_2'} \qquad \frac{\mu\vdash e_2 \;\to\; e_2'}{\mu\vdash \bar{n}>e_2 \;\to\; \bar{n}>e_2'} \qquad \frac{\mu\vdash e_2 \;\to\; e_2'}{\mu\vdash \bar{n}=e_2 \;\to\; \bar{n}=e_2'}$$

$$\frac{n<m}{\mu\vdash \bar{n}<\bar{m} \;\to\; \mathbf{true}} \qquad \frac{n>m}{\mu\vdash \bar{n}>\bar{m} \;\to\; \mathbf{true}} \qquad \frac{n=m}{\mu\vdash \bar{n}=\bar{m} \;\to\; \mathbf{true}}$$

$$\frac{n\geqslant m}{\mu\vdash \bar{n}<\bar{m} \;\to\; \mathbf{false}} \qquad \frac{n\leqslant m}{\mu\vdash \bar{n}>\bar{m} \;\to\; \mathbf{false}} \qquad \frac{n\neq m}{\mu\vdash \bar{n}=\bar{m} \;\to\; \mathbf{false}}$$

$$\frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1 \;\mathbf{or}\; e_2 \;\to\; e_1 \;\mathbf{or}\; e_2} \qquad \frac{}{\mu\vdash \mathbf{true}\;\mathbf{or}\; e_2 \;\to\; \mathbf{true}} \qquad \frac{}{\mu\vdash \mathbf{false}\;\mathbf{or}\; e_2 \;\to\; e_2}$$

$$\frac{\mu\vdash e_1 \;\to\; e_1'}{\mu\vdash e_1 \;\mathbf{and}\; e_2 \;\to\; e_1 \;\mathbf{and}\; e_2} \qquad \frac{}{\mu\vdash \mathbf{true}\;\mathbf{and}\; e_2 \;\to\; e_2} \qquad \frac{}{\mu\vdash \mathbf{false}\;\mathbf{and}\; e_2 \;\to\; \mathbf{false}}$$

$$\frac{\mu\vdash e \;\to\; e'}{\mu\vdash \mathbf{not}\; e \;\to\; \mathbf{not}\; e'} \qquad \frac{}{\mu\vdash \mathbf{not}\;\mathbf{true} \;\to\; \mathbf{false}} \qquad \frac{}{\mu\vdash \mathbf{not}\;\mathbf{false} \;\to\; \mathbf{true}}$$

Grammaire des commandes :

$$\begin{array}{rcl} c & ::= & \mathbf{null} \mid x:=e \mid c;c \mid \mathbf{if}\; e\;\mathbf{then}\; c\;\mathbf{else}\; c\;\mathbf{endif} \\ & \mid & \mathbf{while}\; e\;\mathbf{loop}\; c\;\mathbf{endloop} \mid \mathbf{declare}\; x:\tau:=e\;\mathbf{begin}\; c\;\mathbf{end} \end{array}$$

Grammaire des types :

$$\rho \;::=\; \mathbf{comm}$$

Typage :

$$\frac{\Gamma\vdash e:\mathbf{bool} \quad \Gamma\vdash c_1:\mathbf{comm} \quad \Gamma\vdash c_2:\mathbf{comm}}{\Gamma\vdash \mathbf{if}\; e\;\mathbf{then}\; c_1\;\mathbf{else}\; c_2\;\mathbf{endif}:\mathbf{comm}} \qquad \frac{}{\Gamma\vdash \mathbf{null}:\mathbf{comm}}$$

$$\frac{\Gamma\vdash c_1:\mathbf{comm} \quad \Gamma\vdash c_2:\mathbf{comm}}{\Gamma\vdash c_1;c_2:\mathbf{comm}} \qquad \frac{\Gamma\vdash e:\mathbf{bool} \quad \Gamma\vdash c:\mathbf{comm}}{\Gamma\vdash \mathbf{while}\; e\;\mathbf{loop}\; c\;\mathbf{endloop}:\mathbf{comm}}$$

$$\frac{x:\tau\in\Gamma \quad \Gamma\vdash e:\tau}{\Gamma\vdash x:=e:\mathbf{comm}} \qquad \frac{\Gamma\vdash e:\tau \quad \Gamma,x:\tau\vdash c:\mathbf{comm}}{\Gamma\vdash \mathbf{declare}\; x:\tau:=e\;\mathbf{begin}\; c\;\mathbf{end}:\mathbf{comm}}$$

Sémantique naturelle :

$$\frac{}{\langle \mu, \textbf{null} \rangle \Downarrow \mu} \qquad \frac{\langle \mu, c_1 \rangle \Downarrow \mu'' \quad \langle \mu'', c_2 \rangle \Downarrow \mu'}{\langle \mu, c_1; c_2 \rangle \Downarrow \mu'}$$

$$\frac{\mu \vdash e \Downarrow \textbf{true} \quad \langle \mu, c_1 \rangle \Downarrow \mu'}{\langle \mu, \textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 \textbf{ endif} \rangle \Downarrow \mu'} \qquad \frac{\mu \vdash e \Downarrow \textbf{false} \quad \langle \mu, c_2 \rangle \Downarrow \mu'}{\langle \mu, \textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 \textbf{ endif} \rangle \Downarrow \mu'}$$

$$\frac{\mu \vdash e \Downarrow v}{\langle \mu, x := e \rangle \Downarrow \mu[x \leftarrow v]} \qquad \frac{\mu \vdash e \Downarrow v \quad \langle \mu + (x, v), c \rangle \Downarrow \mu' + (x, v')}{\langle \mu, \textbf{declare } x{:}\,\tau := e \textbf{ begin } c \textbf{ end} \rangle \Downarrow \mu'}$$

$$\frac{}{\langle \mu, \textbf{declare } x{:}\,\tau := e \textbf{ begin null end} \rangle \Downarrow \mu}$$

$$\frac{\mu \vdash e \Downarrow \textbf{false}}{\langle \mu, \textbf{while } e \textbf{ loop } c \textbf{ endloop} \rangle \Downarrow \mu}$$

$$\frac{\mu \vdash e \Downarrow \textbf{true} \quad \langle \mu, c \rangle \Downarrow \mu'' \quad \langle \mu'', \textbf{while } e \textbf{ loop } c \textbf{ endloop} \rangle \Downarrow \mu'}{\langle \mu, \textbf{while } e \textbf{ loop } c \textbf{ endloop} \rangle \Downarrow \mu'}$$

Sémantique opérationnelle structurée :

$$\frac{\mu \vdash e \Downarrow v}{\langle \mu, x := e \rangle \rightarrow \langle \mu[x \leftarrow v], \textbf{null} \rangle}$$

$$\frac{\langle \mu, c_1 \rangle \rightarrow \langle \mu', c_1' \rangle}{\langle \mu, c_1; c_2 \rangle \rightarrow \langle \mu', c_1'; c_2 \rangle} \qquad \frac{}{\langle \mu, \textbf{null}; c_2 \rangle \rightarrow \langle \mu, c_2 \rangle}$$

$$\frac{\mu \vdash e \Downarrow \textbf{true}}{\langle \mu, \textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 \textbf{ endif} \rangle \rightarrow \langle \mu, c_1 \rangle} \qquad \frac{\mu \vdash e \Downarrow \textbf{false}}{\langle \mu, \textbf{if } e \textbf{ then } c_1 \textbf{ else } c_2 \textbf{ endif} \rangle \rightarrow \langle \mu, c_2 \rangle}$$

$$\frac{\mu \vdash e \Downarrow \textbf{false}}{\langle \mu, \textbf{while } e \textbf{ loop } c \textbf{ endloop} \rangle \rightarrow \langle \mu, \textbf{null} \rangle}$$

$$\frac{\mu \vdash e \Downarrow \textbf{true}}{\langle \mu, \textbf{while } e \textbf{ loop } c \textbf{ endloop} \rangle \rightarrow \langle \mu, c; \textbf{while } e \textbf{ loop } c \textbf{ endloop} \rangle}$$

$$\frac{}{\langle \mu, \textbf{declare } x{:}\,\tau := e \textbf{ begin null end} \rangle \rightarrow \langle \mu, \textbf{null} \rangle}$$

$$\frac{\mu \vdash e \Downarrow v \quad \langle \mu + (x, v), c \rangle \rightarrow \langle \mu' + (x, v'), c' \rangle}{\langle \mu, \textbf{declare } x{:}\,\tau := e \textbf{ begin } c \textbf{ end} \rangle \rightarrow \langle \mu', \textbf{declare } x{:}\,\tau := v' \textbf{ begin } c' \textbf{ end} \rangle}$$