# *Two Representations of Music Computed with a Spatial Programming Language*

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**Abstract**

In this long abstract, we show how some musical objects and some algorithmic problems arising in musical theory can be rephrased in spatial terms. This leads to the development of meaningful spatial representations and of efficient spatial programs. The corresponding developments have been implemented in MGS, a rule-based spatial programming language.

*Spatial computing* has proven to be a fruitful paradigm for the (re-)design of algorithms tackling problems embedded in space or having a spatial extension. While music can be seen according to various viewpoints, we propose in this paper a few studies of paradigmatic theoretical music problems from a spatial computing perspective.

Musical theory has received much interest by mathematicians that have found a field to develop algebraic methods to solve, in a very elegant way, problems of enumeration and classification of musical structures. Indeed, the algebraic nature of many musical formalizations has been very early assessed: from the equal temperament to combinatorial properties of the integral serialism and modal techniques. However, the topological properties of those objects have been rarely considered (nevertheless, see [M$^+$02, Tym06]).

Spatial computing distinguishes spaces in computations either as a *resource* or a *result*. In the following, we propose to illustrate each of these two points of view in the field of theoretical music.

**The MGS Programming Language.** MGS is an experimental programming language (see [GM01b, Gia03]) investigating the use of rules to compute with spatial data structures. MGS concepts rely on well established notions in algebraic topology [Mun84] and have been validated through applications in autonomic computing and in the modeling of complex dynamical systems.

In MGS, all data structures are unified under the notion of *topological collection* [GM02]. A topological collection is a *cellular complex* labeled with arbitrary values. The cellular complex acts as a container and the values as the elements of the data structure. A *cellular complex* is a space built by gluing together more elementary spaces called *cells*. A cell is the abstraction of a space with some *dimension*. In a cellular complex, cells are organized following the *incidence relationship*.

Topological collections are transformed using sets of rules called *transformations*. A rule is a pair `pattern => expression`. When a rule is applied on a topological collection, the sub-collections matching with the `pattern` are replaced by the topological collection computed by the evaluation of `expression`. There exists several ways to control the application of a set of rules on a collection but these details are not necessary for the comprehension of the work presented here. A formal presentation of the rewriting mechanism is given in [SMG10].

**Space for Musical Representation.** In this first application, we illustrate how a musical piece can be seen as a computation taking place on specific *resource* space. We focus here on the notion of *Tonnetz* [Coh97, Choa, Chob] and its use for the characterization of tonal music. We show that the topological collection of GBF (Group Based Fields), when carefully defined, makes it possible to describe chords progressions. A GBF [GM01a] is a topological collection that generalizes the usual notion of array, by labeling a cellular complex generated by a mathematical group.

The Neo-Riemannian representation (named after Hugo Riemann, a musicologist) has the ambition to show proximity between notes in terms of consonance instead of chromatism as in the traditional staff. We focus on the strong algebraic strucutre of musical intervals [Lew87] and suggest a spatial representation of a consonance relationship based on Cayley graphs. The properties of the resulting tone networks depend on the definition of consonance. A musical sequence represented in different networks exhibits more or less regularities. We are interested in formalizing these regularities and understanding if a network can characterize signature of a musical piece or style. More, the exploration of tone networks associated to these representations brings elements on scales generation and instruments conception.

**Space as Musical Representation.** A mathematical analysis of music generally relies on the definition of a *model*, that is a mathematical object, whose specific properties describe faithfully some musical characteristics. In the previous case, we followed this approach by defining topological spaces used to exhibit some musical properties. We focus here on the process used to build such a mathematical model of music. Thus, we consider here spaces as *results* of a spatial computing. In particular, we describe a self-assembly mechanism used to define a spatial representation of a tonality based on triadic chords [M$^+$02]. Then, we propose to use this mechanism for the study of a tonality representation based on four-note chords and the analysis of a chord series to F. Chopin.

More detailed applications and other uses of spacial computing for musical analysis are described in [BSM10, BGS11].

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