

Types, Event Structures and the π -Calculus

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Historical perspective

An unfair and myopic view of the last 40 years

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Petri ['60]

Petri nets

Historical perspective

An unfair and myopic view of the last 40 years

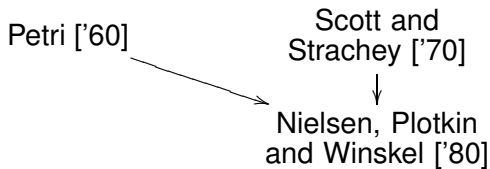
Petri ['60]

Scott and
Strachey ['70]

Denotational semantics - Domain theory

Historical perspective

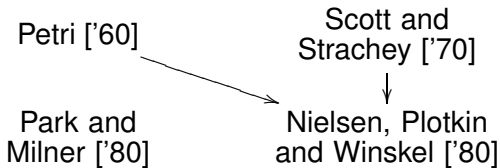
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Event structures

Historical perspective

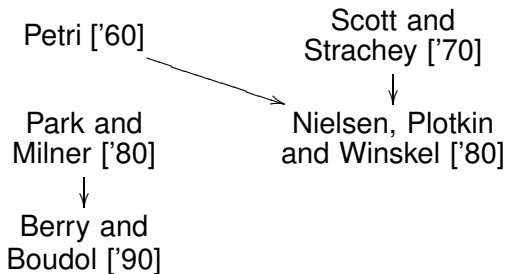
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Transition systems and bisimulation

Historical perspective

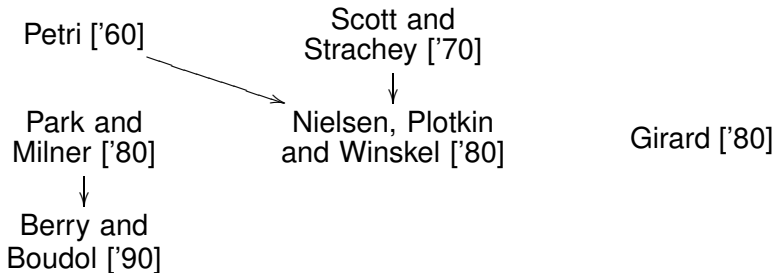
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Reduction semantics

Historical perspective

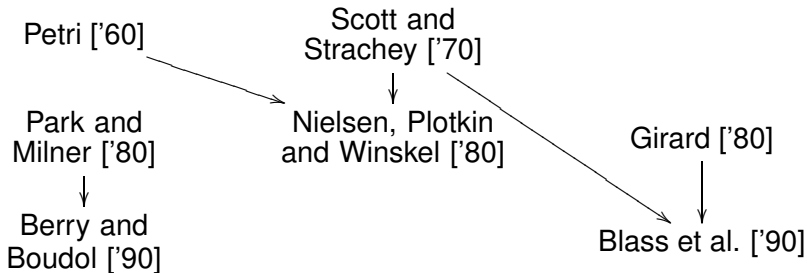
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Linear logic

Historical perspective

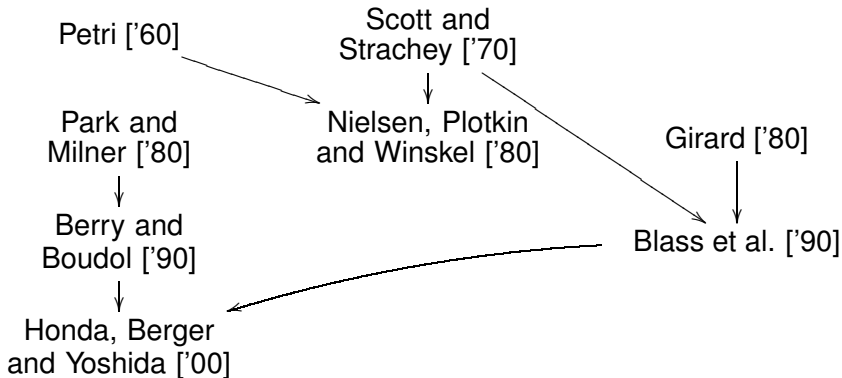
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Game semantics

Historical perspective

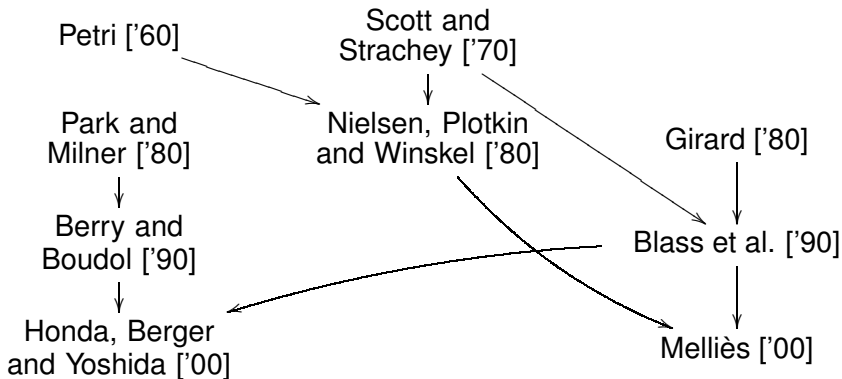
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Linearly typed π calculus

Historical perspective

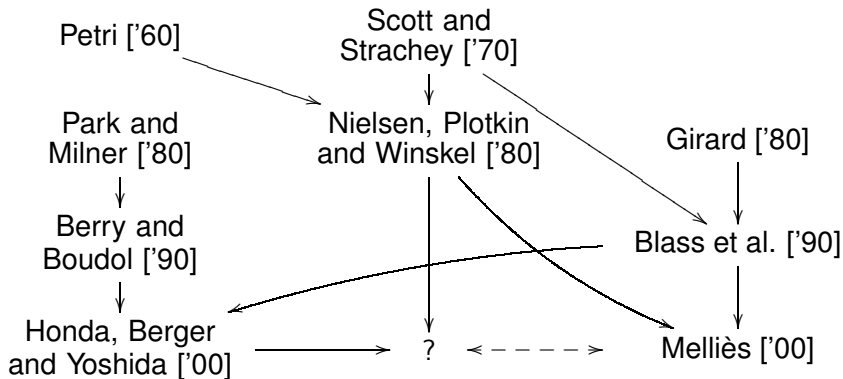
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True concurrent games

Historical perspective

An unfair and myopic view of the last 40 years



We all know what the π -calculus is

$$x(\tilde{y}).P \mid \bar{x}\langle\tilde{z}\rangle.Q \longrightarrow P\{\tilde{z}/\tilde{y}\} \mid Q$$

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bound output only (“internal” mobility)

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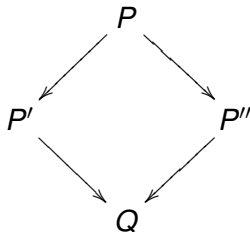
$$x(\tilde{y}).P \mid \bar{x}(\tilde{y}).Q \longrightarrow (\nu \tilde{y})(P \mid Q)$$

We consider a restricted version:
bound output only (“internal” mobility)

A linear type discipline:

- (A) for each linear name there are a unique input and a unique output
- (B) for each replicated name there is a unique stateless replicated input with zero or more dual outputs

Linearly typed π is **confluent**



Road Map

- 1 Event Structures
 - Confusion Freeness
 - Conflict Freeness
- 2 Types
 - Syntax and Semantics
 - Typed Event Structures
- 3 Semantics of π
 - Syntax
 - Event Structure Semantics
 - Correspondence

Road Map

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True concurrency

Standard “interleaving” semantics

- reduces parallelism to nondeterministic interleaving (“expansion law”)
- Labelled transition systems, reduction semantics

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“True concurrent” models

- Represent explicitly causality, conflict, independence
- Petri nets, Mazurkiewicz traces, event structures

Event structures

An event structure is a partial order $\langle E, \leq \rangle$ together with a **conflict** relation \smile

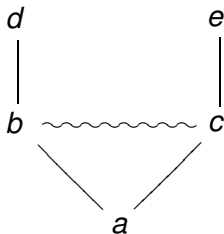
- order represents **causal dependency**
- conflict is irreflexive and symmetric
- conflict is “hereditary”:

$$e_1 \smile e \text{ and } e_1 \leq e_2 \text{ implies } e_2 \smile e$$

A conflict is **immediate** if it is not inherited from another conflict

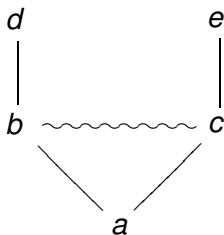
Event structures

Example



Event structures

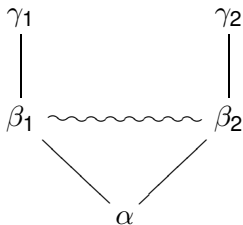
Example



Events can also be labelled: $\lambda : E \rightarrow L$

Event structures

Example



Events can also be labelled: $\lambda : E \rightarrow L$

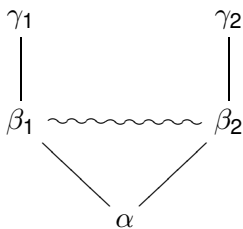
Operators on event structures

Prefixing $\alpha.\mathcal{E}$



Operators on event structures

Prefixing $\alpha.\mathcal{E}$



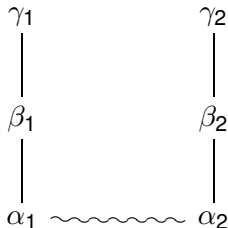
Operators on event structures

Prefix sum $\sum_{i \in I} \alpha_i \cdot \mathcal{E}_i$

$$\begin{array}{c} \gamma_1 \\ | \\ \beta_1 \end{array}$$
$$\begin{array}{c} \gamma_2 \\ | \\ \beta_2 \end{array}$$

Operators on event structures

Prefix sum $\sum_{i \in I} \alpha_i \cdot \mathcal{E}_i$



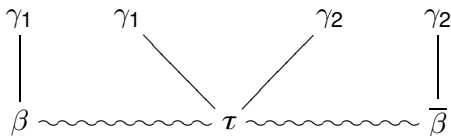
Operators on event structures

Parallel composition $\mathcal{E}_1 \parallel \mathcal{E}_2$

$$\begin{array}{c} \gamma_1 \\ | \\ \beta \end{array}$$
$$\begin{array}{c} \gamma_2 \\ | \\ \overline{\beta} \end{array}$$

Operators on event structures

Parallel composition $\mathcal{E}_1 \parallel \mathcal{E}_2$



A complex construction involving synchronisation

Event structures and transition systems

Consider

- $\mathcal{E} = \langle E, \leq, \smile, \lambda \rangle$, a labelled event structure
- e , one of its minimal events

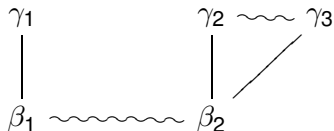
We define $\mathcal{E} \setminus e$ as \mathcal{E} minus event e , and minus all events that are in conflict with e

We can then generate a labelled transition system as follows: if $\lambda(e) = \beta$, then

$$\mathcal{E} \xrightarrow{\beta} \mathcal{E} \setminus e$$

Event structures and transition systems

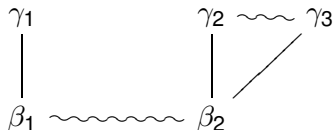
Example



An event structure \mathcal{E}

Event structures and transition systems

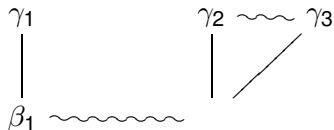
Example



Eliminate a minimal event e (labelled by β_2)

Event structures and transition systems

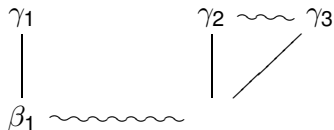
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Event structures and transition systems

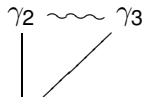
Example



And every event in conflict with it

Event structures and transition systems

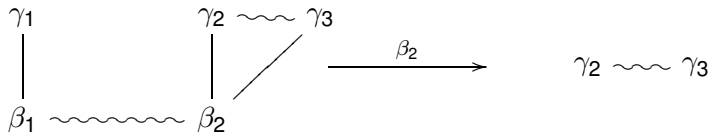
Example



And every event in conflict with it

Event structures and transition systems

Example



$$\mathcal{E} \xrightarrow{\beta_2} \mathcal{E} \upharpoonright \mathbf{e}$$

Confusion freeness

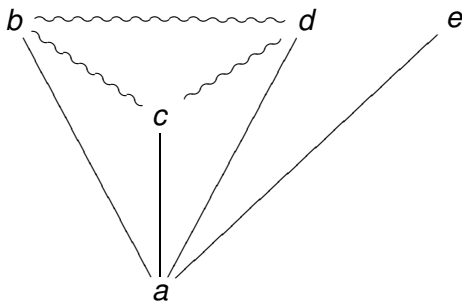
An event structure is **confusion-free** when

- “reflexive” immediate conflict is an equivalence
- any two events in immediate conflict have the same predecessors

The equivalence classes are the **cells**

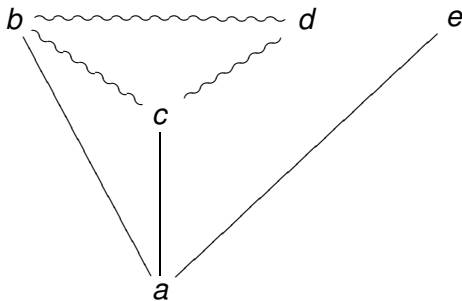
Cells represent local choices

Examples



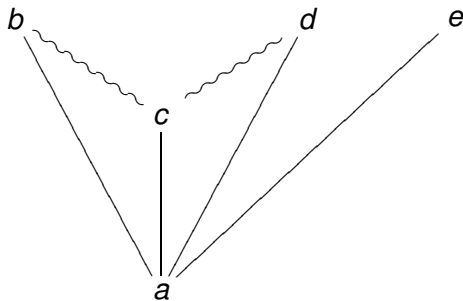
Confusion Free

Examples



Confusion!

Examples



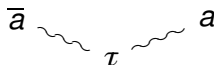
Confusion!

Where confusion arises

Confusion arises from synchronisation

Consider $(\bar{a} \mid a)$

The event structure for this is



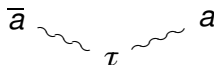
Confusion - the choice is not local

Where confusion arises

Confusion arises from synchronisation

Consider $(\bar{a} \mid a)$

The event structure for this is



Confusion - the choice is not local

Issue: how to perform synchronisation without introducing confusion

Conflict freeness

When the conflict relation is empty, the corresponding transition system is confluent

A special case of confusion freeness

Conflict freeness

When the conflict relation is empty, the corresponding transition system is confluent

A special case of confusion freeness

Idea: give a conflict free event structure semantics to the linear π -calculus

Issues:

- difficult to handle name generation
- hidden conflicts appear

The post office

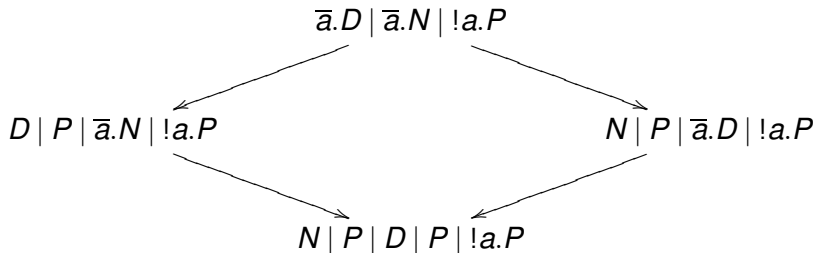
Example:

- Stateless replicated resource: post office $!a.P$
- Clients: customers $\bar{a}.C$

Every customer wants to send a letter a

The post office

The process $\bar{a}.D \mid \bar{a}.N \mid !a.P$ is confluent



The post office

Situation 1: **two customers, one till**

A conflict to resolve: who goes first?

Eventually, it does not matter, but the two events are not independent

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Situation 2: **two customers, infinitely many identical tills**

if the two customers want to go to the same till, there is a conflict

The post office

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Situation 2: **two customers, infinitely many identical tills**

if the two customers want to go to the same till, there is a conflict

Situation 3: **one customer, infinitely many identical tills**

the customer has to choose which till to go to

The post office

Solution: no conflict arises if every possible customer is assigned a specific till **in advance**

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Types for event structures

| | | |
|------------------|---|------------------|
| Γ, Δ | $::= y_1 : \sigma_1, \dots, y_n : \sigma_n$ | type environment |
| τ, σ | $::= \&_{i \in I} \Gamma_i$ | branching |
| | $\oplus_{i \in I} \Gamma_i$ | selection |
| | $\otimes_{i \in I} \Gamma_i$ | server |
| | $\wp_{i \in I} \Gamma_i$ | client |
| | \updownarrow | closed type |

Linearity condition: no name appears more than once

Composing environments

Notion of **matching** of types

- A branching type $\&$ matches the dual selection types \oplus , and the residual type is \updownarrow

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- Two environments Γ_1, Γ_2 be composed if the types of the common names match

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- A branching type $\&$ matches the dual selection types \oplus , and the residual type is \updownarrow
- A server type \otimes matches a client type \wp if all requests correspond to an available resource. The residual is again a server type \otimes that records which resources are still available
- Two environments Γ_1, Γ_2 be composed if the types of the common names match
- Such names are given the residual type by the resulting environment $\Gamma_1 \odot \Gamma_2$

Type environments

Example

- $\tau_1 = \&_{i \in \{1,2\}}(x_i : \bigoplus_{j \in J})$
- $\tau_2 = \bigoplus_{i \in \{1,2\}}(x_i : \&_{j \in J})$
- $\sigma_1 = \wp_{i \in \{1\}}(y_i : \uparrow)$
- $\sigma_2 = \bigotimes_{i \in \{1,2\}}(y_i : \uparrow)$

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- τ_1 matches τ_2
- the residual type is \uparrow

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- σ_1 matches σ_2
- the residual type is $\bigotimes_{i \in \{2\}} (y_i : \uparrow)$

Type environments

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- $\Gamma_1 = a : \tau_1, b : \sigma_1,$
- $\Gamma_2 = a : \tau_2, b : \sigma_2$

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- $\Gamma_2 = a : \tau_2, b : \sigma_2$
- $\Gamma_1 \odot \Gamma_2 = a : \uparrow, b : \bigotimes_{i \in \{2\}} (y_i : \uparrow)$

Labelled event structures

Labels:

| | | | | | | |
|-----------------|-------|--|-----------------|--------|-------|---|
| α, β | $::=$ | $x \text{ in}_i \langle \tilde{y} \rangle$ | branching | τ | $::=$ | $(x, \bar{x}) \text{ in}_i \langle \tilde{y} \rangle$ |
| | | $\bar{x} \text{ in}_i \langle \tilde{y} \rangle$ | selection | | | $(x, \bar{x}) \langle \tilde{y} \rangle$ |
| | | $x \langle \tilde{y} \rangle$ | server | | | |
| | | $\bar{x} \langle \tilde{y} \rangle$ | client | | | |
| | | τ | synchronisation | | | |

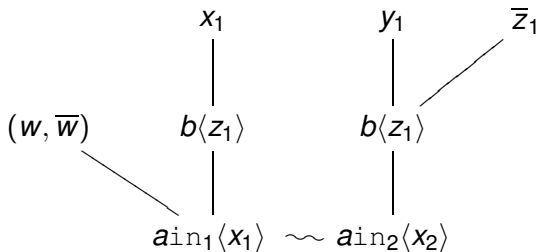
Typing via morphisms

An event structure \mathcal{E} is well typed in Γ if

- \mathcal{E} is confusion free
- cells are partitioned in branching, selection, client, server and synchronisation cells
- all the non-synchronisation events of are represented in Γ
- causality in \mathcal{E} refines name causality of Γ

Technically: via morphisms of the category of event structures

A typed event structure



$$\mathcal{E} \triangleright a : \bigwedge_{i \in \{1,2\}} (x_i : \bigwedge_{k \in \{1\}})$$

$$b : \bigotimes_{j \in \{1\}} (z_j : \bigvee_{l \in \{1\}})$$

Typed event structures

Properties

- Typed event structures are confusion free (by definition)
- Prefixing, prefixed sum and parallel composition preserve typing

In particular **parallel composition of typed event structures is confusion free**

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Theorem [Parallel composition]

If $\mathcal{E}_1 \triangleright \Gamma_1$ and $\mathcal{E}_2 \triangleright \Gamma_2$ and $\Gamma_1 \odot \Gamma_2$ is defined, then

$$(\mathcal{E}_1 \parallel \mathcal{E}_2) \setminus \mathbf{S} \triangleright \Gamma_1 \odot \Gamma_2$$

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$$(\mathcal{E}_1 \parallel \mathcal{E}_2) \setminus S \triangleright \Gamma_1 \odot \Gamma_2$$

(S is the set of names not allowed by the new environment)

Typed event structures

When branching and selection types are trivial:

- Typed event structures are conflict free
- Prefixing, and parallel composition preserve typing

In particular **parallel composition of typed event structures is conflict free**

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The syntax

π processes

| | |
|---|-------------|
| $P ::= x \&_{i \in I} \text{in}_i(\tilde{y}_i).P_i$ | branching |
| $\bar{x} \text{in}_j(\tilde{y}).P$ | selection |
| $!x(\tilde{y}).P$ | server |
| $\bar{x}(\tilde{y}).P$ | client |
| $P \mid Q$ | parallel |
| $(\nu x)P$ | restriction |
| $\mathbf{0}$ | inaction |

The syntax

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| $P ::= x \&_{i \in I} \text{in}_i(\tilde{y}_i).P_i$ | branching |
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The types

π types

$$\begin{array}{lcl}
 \sigma & ::= & \&_{i \in I} (\tilde{\sigma}_i)^\downarrow & \text{branching} \\
 & | & \bigoplus_{i \in I} (\tilde{\sigma}_i)^\uparrow & \text{selection} \\
 & | & (\tilde{\sigma})^\dagger & \text{server} \\
 & | & (\tilde{\sigma})^? & \text{client} \\
 \tau & ::= & \sigma \mid \updownarrow
 \end{array}$$

Environments compose in a similar way as event structure environments

Examples

$$\bar{a}.b \mid \bar{a}.c \mid a$$

This is **not** typable as a appears twice as output

Examples

$$b.\bar{a} \mid c.\bar{b} \mid a.(\bar{c} \mid \bar{e})$$

This is typable since each channel appears at most once as input and output

Examples

$$\bar{a}.(b \oplus c) \mid a.(\bar{d} \& \bar{e})$$

This process is typable, and contains nondeterminism:

$$Q_3 \longrightarrow (b \mid \bar{d})$$

$$Q_3 \longrightarrow (c \mid \bar{e})$$

Examples

$$!b.\bar{a} \mid !b.\bar{c}$$

This is **not** typable as there are two different servers associated with b

Examples

$$!b.\bar{a} \mid \bar{b} \mid !c.\bar{b}$$

This is typable: the two clients on b are associated to a unique server

Typed transition system

Operational semantics

- As usual $P \triangleright \Gamma \xrightarrow{\beta} P' \triangleright \Gamma'$
- The transition must be allowed by the environment
- (The environment performs implicit restrictions)

Event structure semantics of π

The semantics has the form $\llbracket P \triangleright \Gamma \rrbracket^\Delta$, where Δ is an event structure environment

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Δ fixes a choice of the newly generated names

Event structure semantics of π

The semantics has the form $\llbracket P \triangleright \Gamma \rrbracket^\Delta$, where Δ is an event structure environment

Δ fixes a choice of the newly generated names

Δ assigns each client a specific instance of its server

Event structure semantics of π

$$\begin{aligned} & \llbracket \bar{a} \oplus_{i \in I} \text{in}_i(y_i).P_i \triangleright \Gamma, a : \oplus_{i \in I} (\tau_i) \rrbracket^{\Delta, a : \oplus_{i \in I} z_i : \hat{\tau}_i} \\ & = \\ & \sum_{i \in I} \bar{a} \text{in}_i \langle z_i \rangle. \llbracket P_i[z_i/y_i] \triangleright \Gamma, z_i : \tau_i \rrbracket^{\Delta, z_i : \hat{\tau}_i} \end{aligned}$$

Event structure semantics of π

$$\begin{aligned} & \llbracket !a(y).P \triangleright \Gamma, a : (\tau)! \rrbracket^{\Delta, a : \otimes_{k \in K} (y^k : \hat{\tau}^k)} \\ & = \\ & \llbracket_{k \in K} a \langle y^k \rangle . [P[y^k/y] \triangleright \Gamma[y^k/y]] \rrbracket^{\Delta^k, y^k : \hat{\tau}^k} \end{aligned}$$

Event structure semantics of π

$$\begin{aligned} & \llbracket P_1 \mid P_2 \triangleright \Gamma_1 \odot \Gamma_2 \rrbracket^{\Delta_1 \odot \Delta_2} \\ & = \\ & (\llbracket P_1 \triangleright \Gamma_1 \rrbracket^{\Delta_1} \parallel \llbracket P_2 \triangleright \Gamma_2 \rrbracket^{\Delta_2}) \setminus S \end{aligned}$$

Event structure semantics of π

The interpretation functions are partial functions: for the wrong choice of Δ_1, Δ_2 , the interpretation of the parallel composition could be undefined, because $\Delta_1 \odot \Delta_2$ may be undefined

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We perform α -conversion “at compile time”

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Theorem: [Event structure semantics]

For every judgement $P \triangleright \Gamma$ in the π -calculus, there exists an environment Δ such that $\llbracket P \triangleright \Gamma \rrbracket^\Delta$ is defined

Also: $\llbracket P \triangleright \Gamma \rrbracket^\Delta \triangleright \Delta$

Correspondence

Correspondence between transition system and event structure:

Theorem: [Operational correspondence]

If $P \triangleright \Gamma \xrightarrow{\beta} P' \triangleright \Gamma'$, then $\llbracket P \triangleright \Gamma \rrbracket^{\Delta} \xrightarrow{\beta} \cong \llbracket P' \triangleright \Gamma' \rrbracket^{\Delta'}$

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If $\llbracket P \triangleright \Gamma \rrbracket^{\Delta} \xrightarrow{\beta} \mathcal{E}'$, then there exists P' such that $P \triangleright \Gamma \xrightarrow{\beta} P' \triangleright \Gamma'$ and $\llbracket P' \triangleright \Gamma' \rrbracket^{\Delta'} \cong \mathcal{E}'$

What we have done

- First typing system for event structures
- Typing system for true concurrent behavioural properties
- First *explicit* event structure semantics of π

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What we will do

- Probabilistic event structures
- Connections with true concurrent games
- Connections with Beffara's thesis