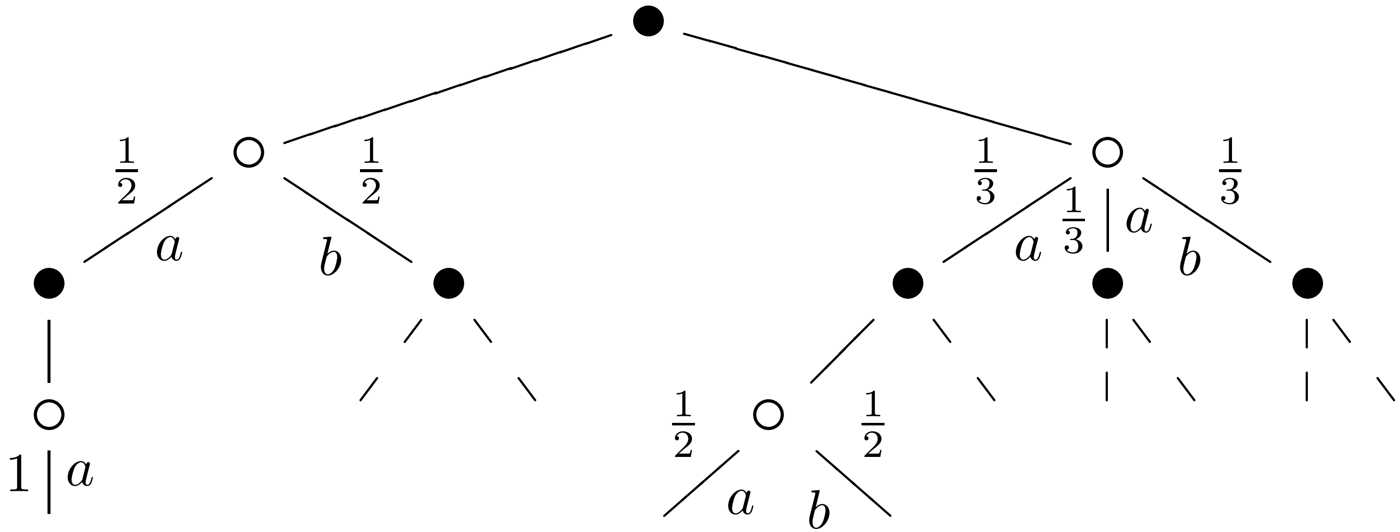
Secret Slides

Markov decision processes

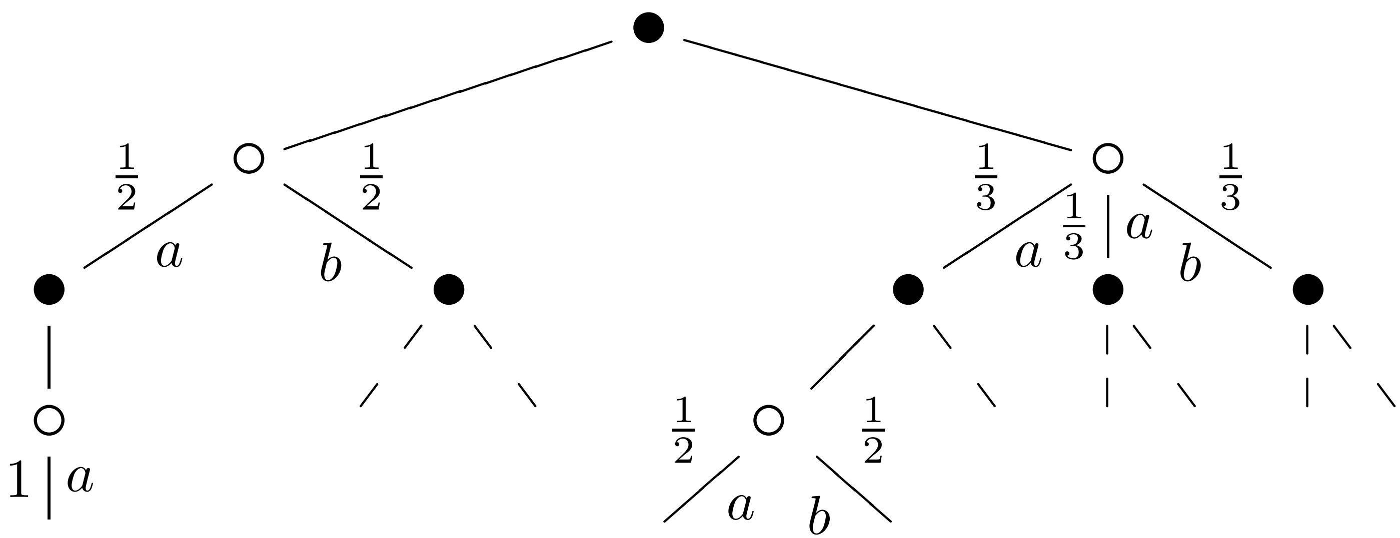
$1\frac{1}{2}$player game (Henzinger, Jurdzinski)



The moves of the full player are called actions

Scheduler

A scheduler is a strategy for the (full) player



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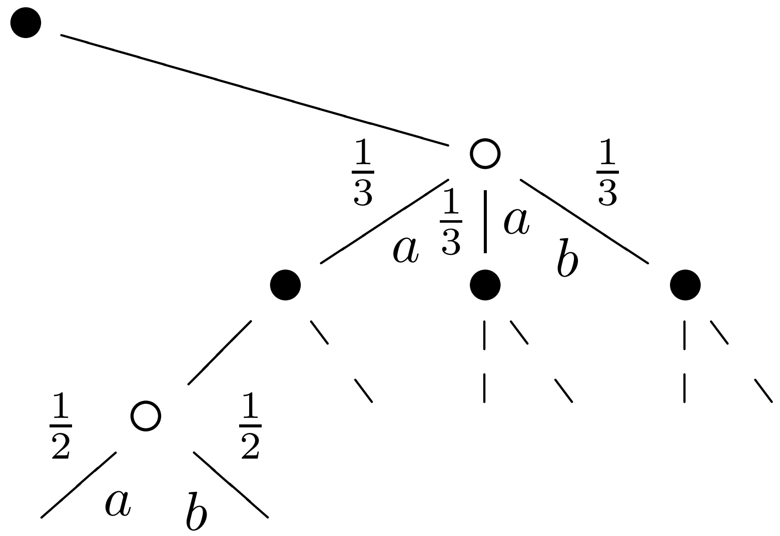

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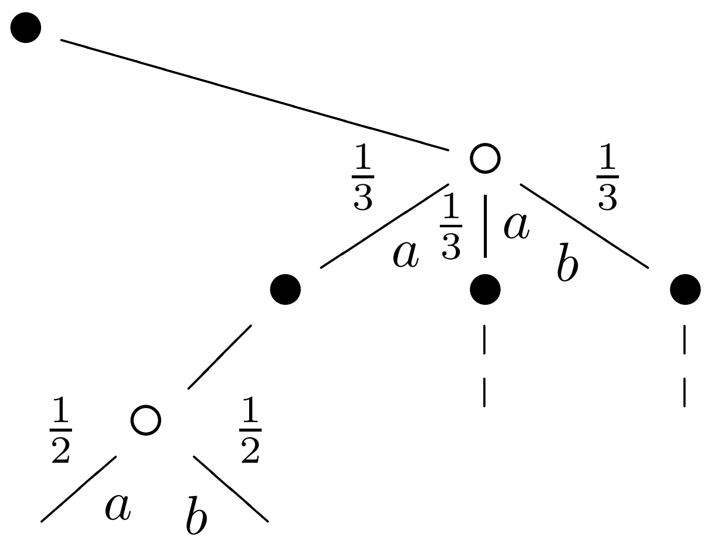

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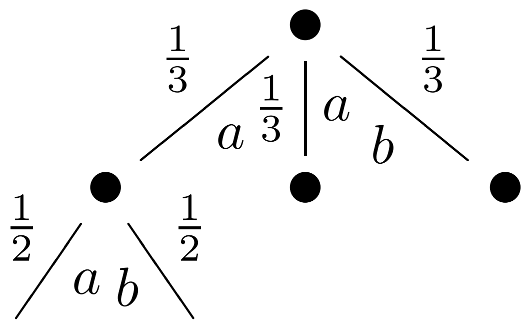

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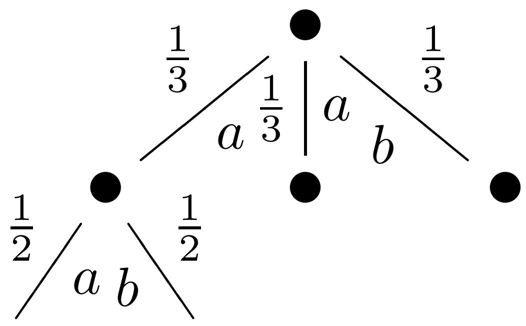


A scheduler leaves us with a (labelled) Markov process

Runs

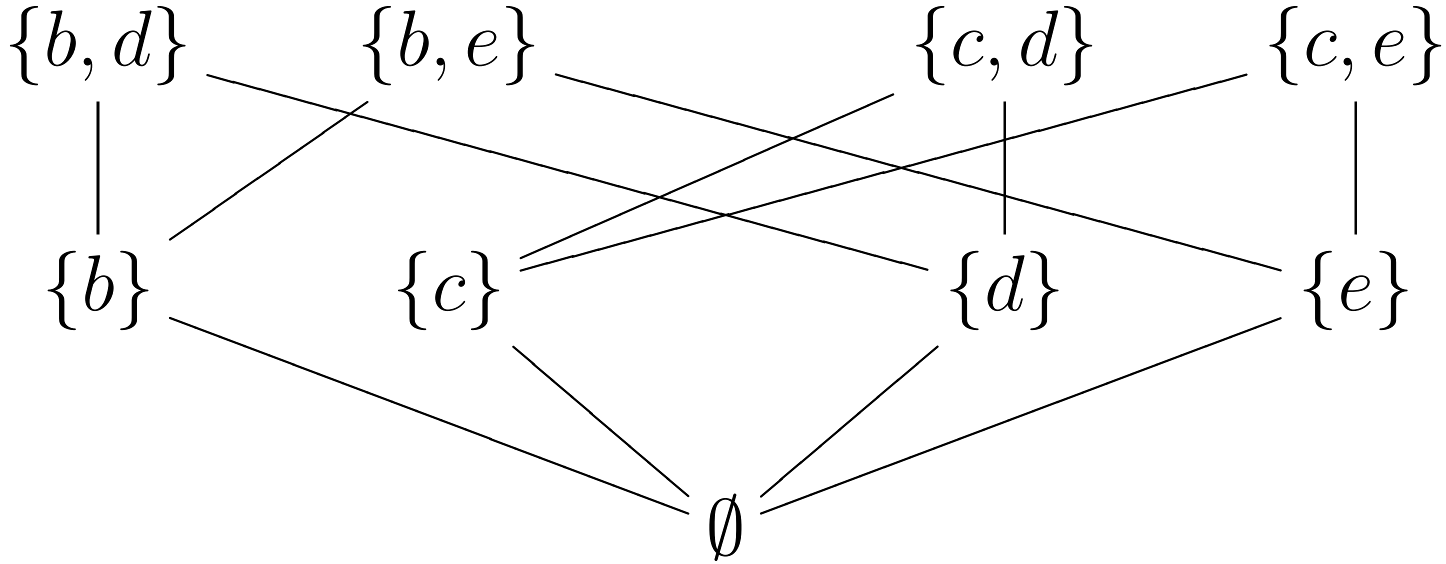
- A finite run of a Markov decision process is a probability distribution over strings of the same length (DeAlfaro, Henzinger, Jhala)
- The set of maximal runs is equipped with a measure (Segala)
- Probabilistic verification uses interleaving
- Temporal logics talk about schedulers

Valuations and independence

Not *all* continuous valuations are obtained the way we have seen

$$b \leadsto c \qquad d \leadsto e$$

This is $\mathcal{L}(\mathcal{E})$:



Valuations and independence

Now put

- $\xi(\emptyset\uparrow) = 1$
- $\xi(\{b\}\uparrow) = \xi(\{c\}\uparrow) = \xi(\{d\}\uparrow) = \xi(\{e\}\uparrow) = 1/2$
- $\xi(\{b,d\}\uparrow) = \xi(\{c,e\}\uparrow) = 0$
- $\xi(\{b,e\}\uparrow) = \xi(\{c,d\}\uparrow) = 1/2$

No valuation on $\mathcal{E}$ generates ξ

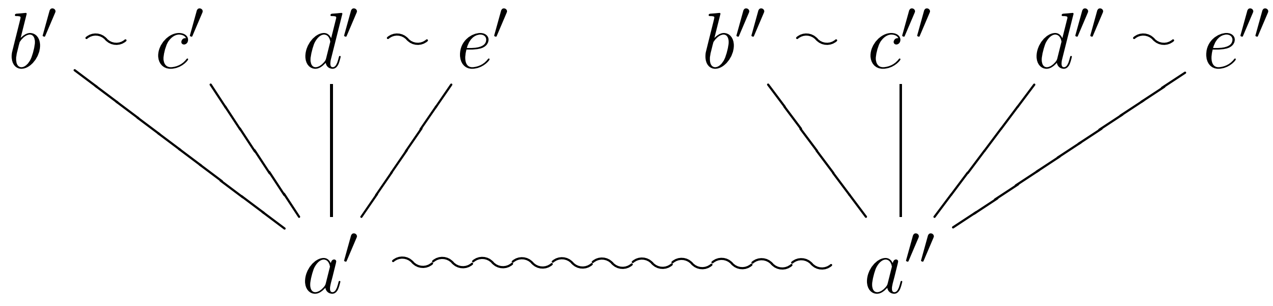
There is a correlations between the two cells

Morphisms

A morphism $f : \mathcal{E} \to \mathcal{E}'$ is a (partial) function from $E \to E'$ such that if x is a configuration, $f(x)$ is a configuration

The role of morphisms

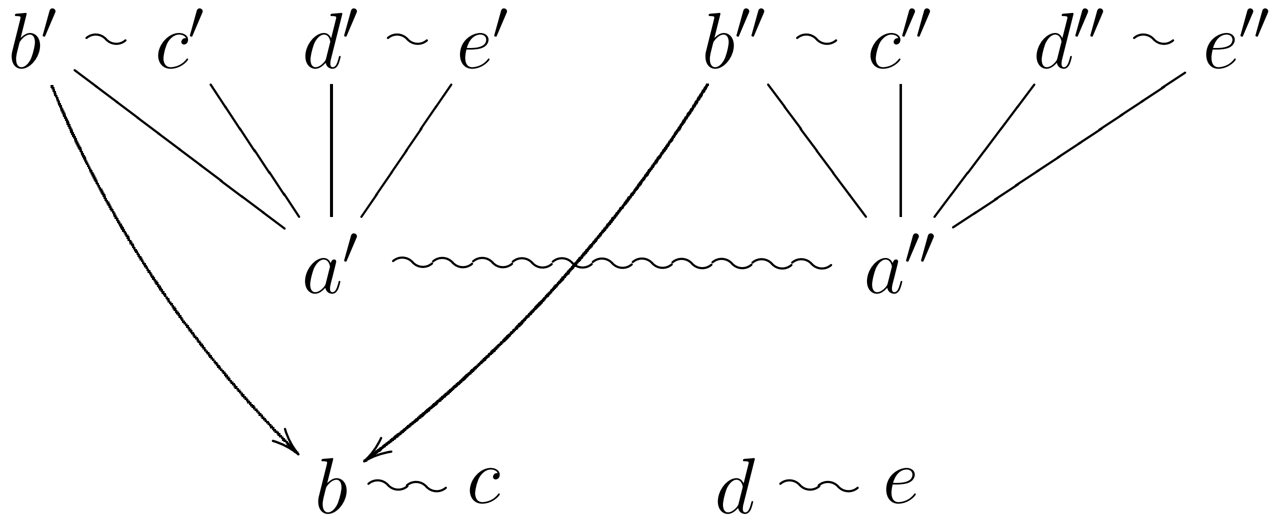
How we get valuations without independence



$b \sim\sim c \qquad d \sim\sim e$

The role of morphisms

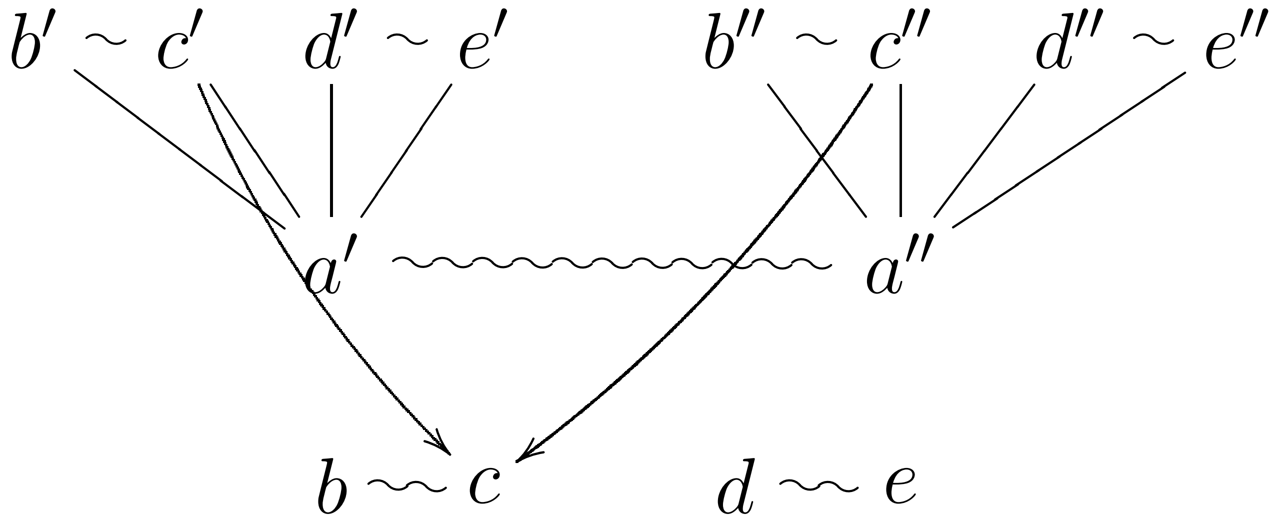
How we get valuations without independence



A morphism $f : \mathcal{E} \to \mathcal{E}'$

The role of morphisms

How we get valuations without independence



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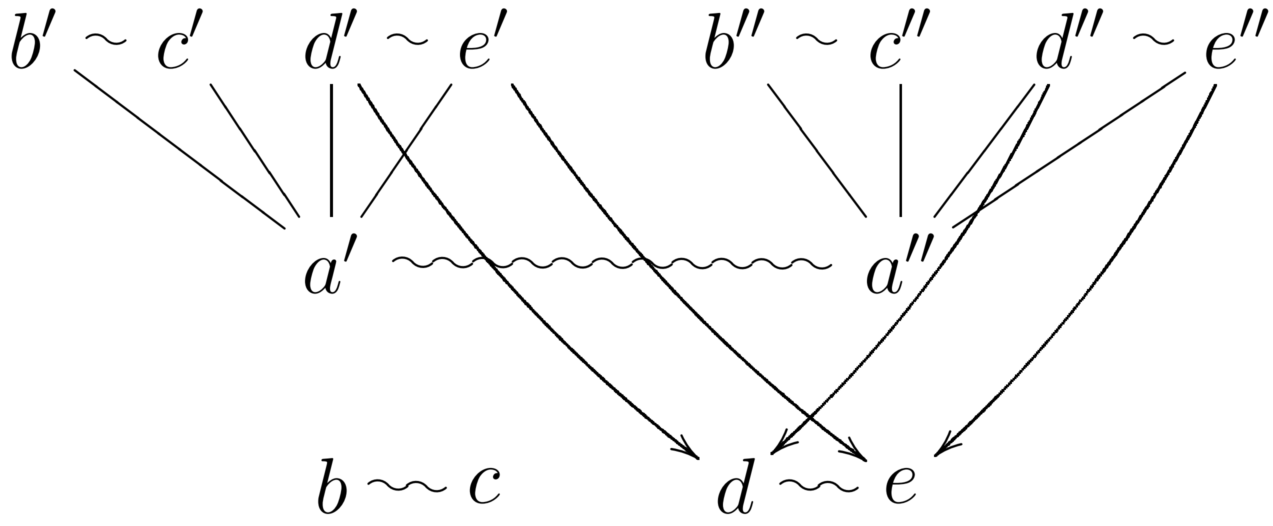

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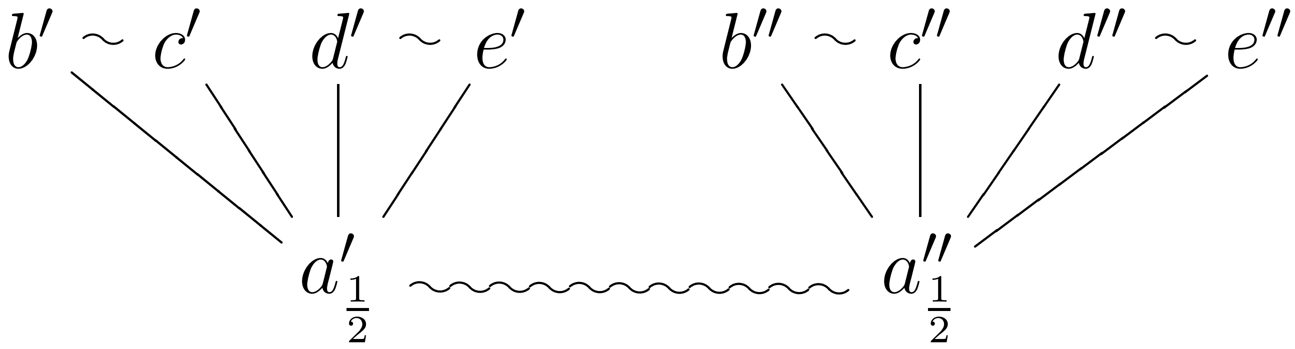
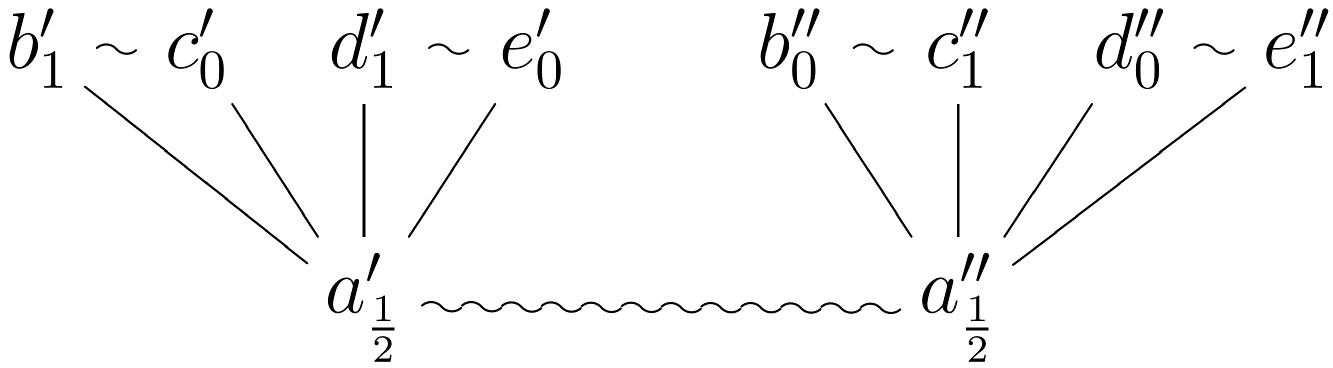


$$b \approx c \qquad d \approx e$$

A valuation ν on $\mathcal{E}$

The role of morphisms

How we get valuations without independence

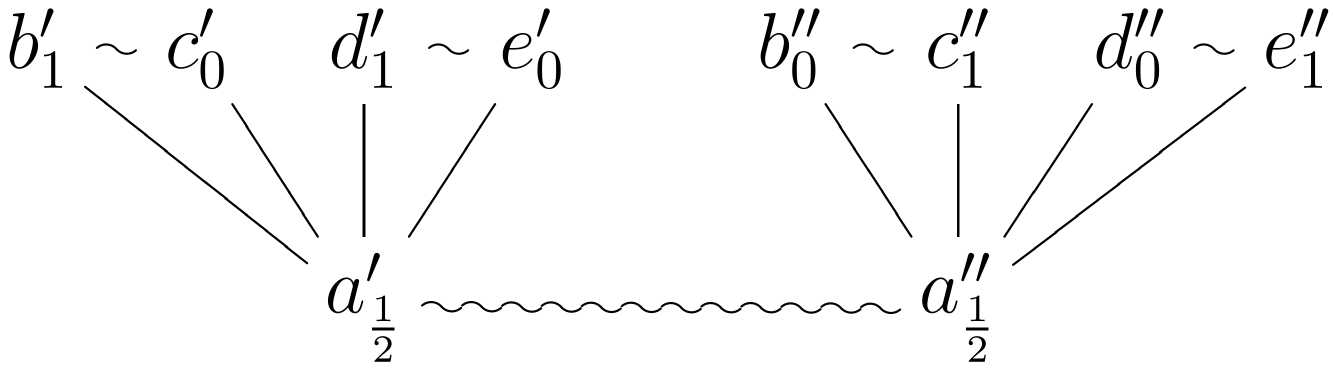


$$b \sim\!\sim c \qquad d \sim\!\sim e$$

A valuation ν on $\mathcal{E}$

The role of morphisms

How we get valuations without independence

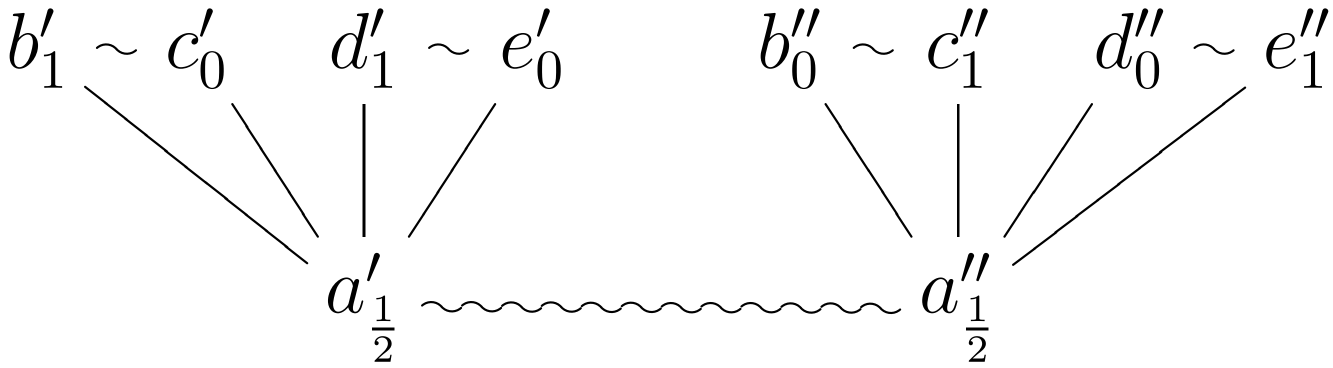


$$b \rightsquigarrow c \qquad d \rightsquigarrow e$$

Define a valuation ν' on $\mathcal{E}'$ by pulling back ν

The role of morphisms

How we get valuations without independence

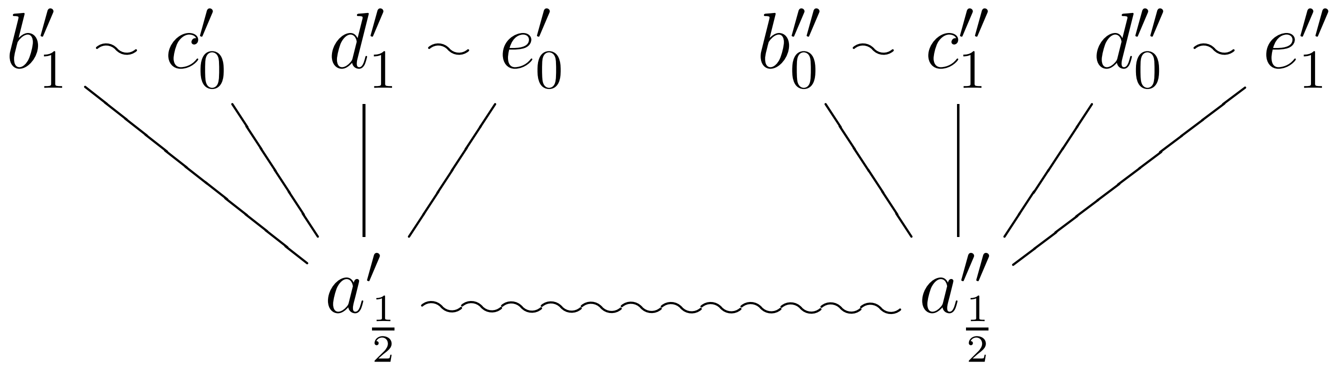


$$b \sim\!\sim c \qquad\qquad d \sim\!\sim e$$

$$\nu'(y) = \sum_{f(x)=y} \nu(x)$$

The role of morphisms

How we get valuations without independence

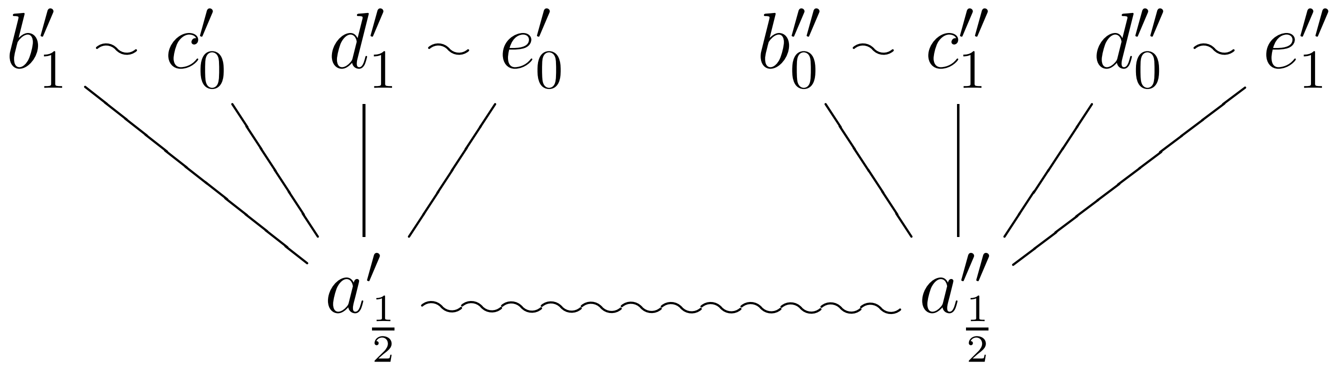


$$b \rightsquigarrow c \qquad d \rightsquigarrow e$$

$$\nu'(\{b\}) = \frac{1}{2}, \nu'(\{e\}) = \frac{1}{2}$$
$$\nu'(\{b, e\}) = 0$$

The role of morphisms

How we get valuations without independence



$$b \rightsquigarrow c \qquad d \rightsquigarrow e$$

Negative correlation between b and e

Due to a hidden choice

New definition

A valuation on $\mathcal{E}$ is a function $v : \mathcal{L}_{fin}(E) \rightarrow [0,1]$ such that for every test C

$$\sum_{x \in C} v(x) = 1$$

Theorem

For every valuation v on an event structure $\mathcal{E}$ there is a unique continuous valuation ν_v on $\mathcal{L}(\mathcal{E})$ such that, for every finite configuration x:

$$\nu_v(x \uparrow) = v(x)$$

Valuations and partiality

We have to consider partial probability distributions

- $\xi(\emptyset \uparrow) = 1$
- $\xi(\{a\} \uparrow) = \xi(\{b\} \uparrow)$
  $= \xi(\{c\} \uparrow) = \xi(\{d\} \uparrow) = 1/3$
- $\xi(\{a, c\} \uparrow) = \xi(\{b, d\} \uparrow)$
  $= \xi(\{a, d\} \uparrow) = \xi(\{b, c\} \uparrow) = 1/9$

Some probability "leaks"

Leaking valuations

$$b_{\frac{1}{3}} \sim c_{\frac{1}{3}} \qquad d_{\frac{1}{3}} \sim e_{\frac{1}{3}}$$

The leaking probability can be accounted for

Leaking valuations

$$\partial_{\frac{1}{3}} \sim b_{\frac{1}{3}} \sim c_{\frac{1}{3}} \qquad d_{\frac{1}{3}} \sim e_{\frac{1}{3}} \sim \partial_{\frac{1}{3}}$$

The leaking probability can be accounted for by invisible events, representing deadlock